Numerical search of Classification Problems in one dimensional Cellular Automata

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Notation and Basic Facts

We consider the **one-dimensional, binary cellular automata (CA) on finite lattices with periodic boundary conditions**. We assume that the size of the neighbourhood extends over a distance r from a cell being considered, known as the range or radius of the neighbourhood, if every node depends on $2r+1$ neighbors, we define:

- A local transition function (LTF): $f : \{0,1\}^{2r+1} \rightarrow \{0,1\}$.
- A global transition function: $F = (f_1, .., f_L) : \{0,1\}^{2r+1} \rightarrow \{0,1\}^{2r+1}$ with f_i LTF, $i \in \{1, \dots L\}$.
- Fixed Point: is a configuration $Y \in \{0,1\}^{2r+1}$ such that $F(Y) = Y$.
- Limit Cycle: is a sequence of configurations $x^0 \dots x^{p-1}$, $p > 1$, such that:
  -$x^j \in \{0,1\}^{2r+1}$ and the x^j are pairwise distinct.
  -$F(x^j) = x^{j+1}, \forall j = 0, \dots, p-2$.
  -$F(x^{p-1}) = x^0$.

Notation	Meaning
1^* (0^*)	$1^* \equiv \{1\}^*$ $(0^* \equiv \{0\}^*)$
$\|x\|_w$	Number of occurrences of w in the configuration x with periodic boundary, where $x, w \in \{0,1\}^*$. E.g., if $x = 10100$ and $w = 01$ then $\|x\|_w = 2$
$\|x\|_1$ $(\|x\|_0)$	Number of 1s (0s) in the configuration x
$\|x\|$	The size of configuration x, i.e., $\|x\| \equiv \|x\|_0 + \|x\|_1$
S_{all}	$S_{all} \equiv \bigcup_{n \geq 2r+1} \{0,1\}^n$
S_{odd}	$S_{odd} \equiv \bigcup_{n \geq 2r+1,\ n\ odd} \{0,1\}^n$
S_{even}	$S_{even} \equiv \bigcup_{n \geq 2r+1,\ n\ even} \{0,1\}^n$

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Classification Problem (CP)

We define a **Classification Problem (CP)** as a 5-tuple $(S, \mathcal{P}, \mathcal{Q}, \mathcal{A}_{\mathcal{P}}, \mathcal{A}_{\mathcal{Q}})$, consisting of

$$CP : \begin{cases} \text{A } \textit{definition set } S \subseteq \{0,1\}^* \\ \text{A } \textit{property set } \mathcal{P} \subseteq S \\ \text{The } \textit{complementary property set } \mathcal{Q} \equiv S \backslash \mathcal{P} \text{ (i.e., } S = \mathcal{P} \cup \mathcal{Q}) \\ \text{The } \textit{classification sets } \mathcal{A}_{\mathcal{P}} \subseteq \mathcal{P} \text{ and } \mathcal{A}_{\mathcal{Q}} \subseteq \mathcal{Q} \end{cases}$$

with the goal of deciding whether there exists a CA A of radius r and periodic boundary conditions, such that

(a) If $x \in \mathcal{P}$, then x should converge to an arbitrary $u \in \mathcal{A}_{\mathcal{P}}$

(b) If $x \in \mathcal{Q}$, then x should converge to an arbitrary $v \in \mathcal{A}_{\mathcal{Q}}$

for all $x \in S$, with $|x| \geq \lfloor 2r + 1 \rfloor$, and where the elements of $\mathcal{A}_{\mathcal{P}}$ and $\mathcal{A}_{\mathcal{Q}}$ must be attractors of the CA A (i.e., limit cycles or fixed points). If such a CA exists, we say that the CP *has solution*.

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Density Classification Problem

The CA performing the Density Problem should converge to a fixed point of all 1's if the initial configuration contains more 1's than 0's, and to a fixed point of all 0's if the initial configuration contains more 0's than 1's. This is:
Density Classification Problem – DCP

$$DCP : \begin{cases} S = S_{odd} \\ \mathcal{P} = \{x \in S_{odd} : |x|_1 > |x|_0\} \\ \mathcal{Q} \equiv S_{odd} \backslash \mathcal{P} = \{x \in S_{odd} : |x|_1 < |x|_0\} \\ \mathcal{A}_{\mathcal{P}} = 1^* \cap S_{odd} \text{ and } \mathcal{A}_{\mathcal{Q}} = 0^* \cap S_{odd} \end{cases}$$

In [7] it shown that no one-dimensional, binary elementary cellular automaton which classifies binary strings according to their densities of 1's and 0's can be constructed. But with a pair of elementary rules, namely the "traffic rule" 184 and the "majority rule" 232, performs the task perfectly.

Example

$$\begin{array}{ccc} 1\,1\,0\,0\,1\,1\,0\,1\,1 & & 0\,1\,0\,0\,1\,1\,0\,1\,0 \\ \downarrow & & \downarrow \\ 1\,1\,1\,1\,1\,1\,1\,1\,1 & & 0\,0\,0\,0\,0\,0\,0\,0\,0 \end{array}$$

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Parity Classification Problem – ParCP

We consider the parity problem: if the initial configuration contains an odd number of 1s, the lattice should converge to all 1s; otherwise, it should converge to all 0s.

$$ParCP : \begin{cases} S = S_{odd} \\ \mathcal{P} = \{x \in S_{odd} : |x|_1 \text{ is odd}\} \\ \mathcal{Q} \equiv S_{odd} \backslash \mathcal{P} = \{x \in S_{odd} : |x|_1 \text{ is even}\} \\ \mathcal{A}_{\mathcal{P}} = 1^* \cap S_{odd} \text{ and } \mathcal{A}_{\mathcal{Q}} = 0^* \cap S_{odd} \end{cases}$$

In [3] it was proven there exists no radius 1 y 2 rule that can solve the parity problem from arbitrary initial configurations.

In [3] it was designed a radius 4 rule that solves the problem of parity

Whether or not there exists a radius 3 rule that solves the parity problem remains an open problem.

Example

$$\begin{array}{c} 0\,1\,0\,0\,1\,0\,0\,1\,0 \\ \downarrow \\ 1\,1\,1\,1\,1\,1\,1\,1\,1 \end{array} \qquad \begin{array}{c} 0\,1\,0\,0\,1\,1\,0\,1\,0 \\ \downarrow \\ 0\,0\,0\,0\,0\,0\,0\,0\,0 \end{array}$$

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Classification Problems: CA study that have only fixed points $\overrightarrow{0}$ and $\overrightarrow{1}$.

In this section we study the CP that only have fixed points $\overrightarrow{0}$ and $\overrightarrow{1}$ for any initial configuration, i.e.

$$CP1 : \begin{cases} S = S_{all} \\ \text{A } \textit{property set } \mathcal{P} \subseteq S \\ \text{The } \textit{complementary property set } \mathcal{Q} \equiv S \backslash \mathcal{P} \text{ (i.e., } S = \mathcal{P} \cup \mathcal{Q}) \\ \mathcal{A}_{\mathcal{P}} = 1^* \cap S_{all} \text{ and } \mathcal{A}_{\mathcal{Q}} = 0^* \cap S_{all} \end{cases}$$

We consider CA of radius $r = 1$ and $r = 1.5$.

To determine the CP1 first we give some basic propositions to be satisfies.

Proposition 3.1

If f solve the clasification problem (CP1), then $f(0, 0 \cdots, 0) = 0$ and $f(1, 1 \cdots, 1) = 1$.

Proposition 3.2

If f solve the CP1, then for any configuration $X \notin \{\overrightarrow{0}, \overrightarrow{1}\}$, $F(X) \neq X$, i.e. X is not fixed point.

Proof. Let $X \notin \{\overrightarrow{0}, \overrightarrow{1}\}$, if $F(X) = X \Rightarrow F^2(X) = F(X) = X \Rightarrow F^p(X) = X$, $\forall p \in \mathbb{Z}^+$, therefore the CA converges to $X \notin \{\overrightarrow{0}, \overrightarrow{1}\}$.

Proposition 3.3

If f solve the CP1, then for any configuration $X \notin \{\overrightarrow{0}, \overrightarrow{1}\}$ it has no limit cycle.

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CA of radius r $= 1$

The function $f : \{0,1\}^3 \to \{0,1\}$ such that

$$x_i^{t+1} = f(x_{i-1}^t, x_i^t, x_{i+1}^t)$$

it will be called an elementary cellular automata. In [10] rule lists the $2^{2^3} = 256$ possible choices of f. Only 88 of these rules are discussed, since the remaining are equivalent to any of these. 24 of the 88 rules satisfy Property 3.1. Of these, we remove all rules that does not meet Properties 3.2 and 3.3. Two of these rules finally converge only to $\overrightarrow{0}$ and $\overrightarrow{1}$, these are:

1.- $f(x_1, x_2, x_3) = x_1 x_2 x_3$

2.- $f(x_1, x_2, x_3) = x_1 x_2$

Notation: : We denote by X^0 the initial configuration and
$X^t = F^t(X^0) \equiv \underbrace{F \circ \cdots \circ F(X^0)}_{t \text{ times}}$

Proposition 3.4

If $f(x_1, x_2, x_3) = x_1 x_2 x_3$, $x_1, x_2, x_3 \in \{0,1\}$ then any CA of lattice size $n \geq 3$ with periodic boundary conditions converges to $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$. If $X^0 = \overrightarrow{1}$ the CA converges to $\overrightarrow{1}$.

Proof. Let $X^0 \neq \overrightarrow{1}$ then X^0 contains at least one 0

$$X^0 = * \ * \ 0 \ * \ * \ * \ *$$
$$\Rightarrow X^1 = * \, 0 \, 0 \, 0 \ * \ * \, *$$
$$\Rightarrow X^1 = 0 \, 0 \, 0 \, 0 \, 0 \ * \ *$$

Hence 0 is increased and so the CA converges to $\overrightarrow{0}$. If $X^0 = \overrightarrow{1}$ the CA converges to $\overrightarrow{1}$ for Property 3.1.

Proposition 3.5

If $f(x_1, x_2, x_3) = x_1 x_2$, $x_1, x_2, x_3 \in \{0, 1\}$ then any CA of size $n \geq 3$ with periodic boundary conditions converges to $\overrightarrow{0}$, $\forall\, X^0 \neq \overrightarrow{1}$. If $X^0 = \overrightarrow{1}$ the CA converges to $\overrightarrow{1}$.

Proof. Let $X^0 \neq \overrightarrow{1}$ then X^0 contains at least one 0

$$X^0 = * \ * \ 0 \ * \ * \ * \ *$$
$$\Rightarrow X^1 = * \ * \ 0 \, 0 \ * \ * \, *$$
$$\Rightarrow X^1 = * \ * \ 0 \, 0 \, 0 \ * \ *$$

Hence 0 is increased and so the CA converges to $\overrightarrow{0}$. If $X^0 = \overrightarrow{1}$ the CA converges to $\overrightarrow{1}$ for Property 3.1.

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CA of radius r = 1.5

We study the CA with local transition function $f : \{0,1\}^4 \rightarrow \{0,1\}$ defined by

$$x_i^{t+1} = f(x_{i-2}^t, x_{i-1}^t, x_i^t, x_{i+1}^t).$$

There exist $2^{16} = 65536$ possible options for f.

We want to determine all the CA that meeting the CP1 for all lattice size. To find these CP1, criteria are presented and therefore we discard the CA that does not meet these conditions.

Remark: By Proposition 3.1 we remove all the CA such that $f(0,0,0,0) = 1$ and $f(1,1,1,1) = 0$. Rules that satisfies the Proposition 3.1 are 16384.

In the following we will consider only the CA that satisfies the Proposition 3.1.

By Proposition 3.2 we remove the CA of lattice size $n = 4,5,6,7$ that $F(X) = X$, $\forall X \in \{0,1\}^4 \setminus \{\vec{0}, \vec{1}\}$.

Example: If $X^0 = 1000$ we remove the CA that satisfies $X^1 = 1000$, i.e. the configurations such that $f(1,0,0,0) = 0 \wedge f(0,0,0,1) = 0 \wedge f(0,0,1,0) = 1 \wedge f(0,1,0,0) = 0$. 1024 of the 16384 rules were eliminated by criterion.

Proposition 3.6

Consider a CA of lattice size n. If $X^p = x_1x_2x_3\ldots x_n$ and $X^{p+1} = x_2x_3\ldots x_nx_1$, then the CA has limit cycle.

Proof. Let $X^0 \notin \{\overrightarrow{0}, \overrightarrow{1}\}$, such that:

$$\begin{aligned} X^0 = x_1x_2\ldots x_n \wedge X^1 = x_2\ldots x_nx_1 &\Rightarrow X^2 = x_3\ldots x_nx_1x_2 \\ &\Rightarrow X^3 = x_4\ldots x_nx_1x_2x_3 \\ &\vdots \\ &\Rightarrow X^{n-1} = x_nx_1\ldots x_{n-1} \\ &\Rightarrow X^n = x_1x_2\ldots x_n \end{aligned}$$

therefore the CA has the limit cycle.

We remove the CA of lattice size $n = 4, 5, 6, 7$ such that satisfies the Proposition 3.6 (with $X^0 \notin \{\overrightarrow{0}, \overrightarrow{1}\}$).

Example: If $X^0 = 1000$ we remove the CA that satisfies $X^1 = 0100$, i.e. the configurations such that $f(1,0,0,0) = 0 \wedge f(0,0,0,1) = 0 \wedge f(0,0,1,0) = 0 \wedge f(0,1,0,0) = 1$.

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Analysis of the CP (r=1.5)

From the proposals mentioned above and the development of algorithms, it came to 138 rules (functions) candidates for classification problems. We analyze 69 of these rules, since the others are equivalent to these.

Through the following propositions we test the convergence to $\overrightarrow{0}$ and $\overrightarrow{1}$ of most rules found.

Proposition 3.7

If $f(0,a,b,c) = 0 \wedge f(d,0,e,f) = 0, \forall\ a,b,c,d,e,f \in \{0,1\}$ then any CA of size $n \geq 4$ converges to $\overrightarrow{0}, \forall\ X^0 \neq \overrightarrow{1}$.

Proof. Let $X^0 \neq \overrightarrow{1}$, i.e. X^0 it contains at least one 0, then

$$\begin{aligned} X^0 &= * \ 0 \ * \ * \ * \ * \\ \Rightarrow\ X^1 &= * \ * \ 0 \ 0 \ * \ * \\ \Rightarrow\ X^2 &= * \ * \ * \ 0 \ 0 \ 0 \end{aligned}$$

Therefore the number of zeros increases in each time step, so the CA converges to $\overrightarrow{0}$ in less than n iterations.

Remark: The rules that satisfies the proposition 3.7 are: 32768, 49152, 40960, 57344, 36864, 53248, 45056, 61440.

Proposition 3.8

If $f(0,a,b,c) = 0 \wedge f(d,e,0,1) = 0 \wedge f(1,0,f,0) = 0 \wedge f(g,0,0,h) = 0$, $\forall\ a,b,c,d,e,f,g,h \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Proof. Suppose $X^0 \notin \{\overrightarrow{0}, \overrightarrow{1}\}$, i.e. X^0 It contains at least 01,

$$
\begin{aligned}
& X^0 = * \ * \ 0\ 1 \ * \ * \ * \ * * \\
\Rightarrow\ & X^1 = * \ * \ 0 \ * \ 0 \ * \ * \ * \ *, \quad \text{then } f(0,a,b,c) = 0 \ \wedge f(d,e,0,1) = 0 \\
\Rightarrow\ & X^2 = * \ * \ * \, 0\ 0 \ * \ 0 \ * \ *, \quad \text{then } f(*,0,f,0) = 0 \\
\Rightarrow\ & X^3 = * \ * \ * \ * \ 0\ 0\ 0 \ * \ 0
\end{aligned}
$$

Therefore the number of zeros increases in each time step, so the CA converges to $\overrightarrow{0}$ in less than n iterations.

Remark: The rules that satisfies the Proposition 3.8 are: 34816, 51200, 38912, 5529.

Proposition 3.9

If $f(a,b,a,b) = 1 \wedge f(0,0,c,d) = 0 \wedge f(e,0,0,f) = 0 \wedge f(0,1,1,g) = 0 \wedge f(h,0,1,1) = 0 \wedge f(i,j,0,0) = 0$, $\forall\ a,b,c,d,e,f,g,h,i,j \in \{0,1\}$ *y* $a \neq b$ *then any CA of odd lattice sized* n, $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$. *If* n *is wide lattice size, then the CA converges to* $\overrightarrow{0}$, $X^0 \notin \{\overrightarrow{1}, 01^{\frac{n}{2}}\}$, *otherwise* $(X^0 \in \{\overrightarrow{1}, 01^{\frac{n}{2}}\})$ *the CA converges to* $\overrightarrow{1}$.

Proof.

1. Suppose that $X^0 \notin \{\overrightarrow{0}, \overrightarrow{1}\}$, odd lattice sized n i.e. X^0 contains at least yxx with $y, x \in \{0,1\}$, $x \neq y$. For this we have two cases:

   Case 1

$$\begin{aligned}
& X^0 = * \ * \ 1\,0\,0 \ * \ * \ * \ * * \\
\Rightarrow \ & X^1 = * \ * \ * \,0\,0\,0 \ * \ * \ * \ *, \quad f(0,0,c,d)=0 \ \wedge f(e,0,0,f)=0 \ \wedge f(i,j,0,0)=0 \\
\Rightarrow \ & X^2 = * \ * \ * \,0\,0\,0\,0 \ * \ * *, \quad f(0,0,c,d)=0 \ \wedge f(e,0,0,f)=0 \ \wedge f(i,j,0,0)=0
\end{aligned}$$

   Then 0 is increasing, therefore the CA converges to $\overrightarrow{0}$.

   Case 2

$$\begin{aligned}
& X^0 = * \ * \ 0\,1\,1 \ * \ * \ * \ * * \\
\Rightarrow \ & X^1 = * \ * \ * \,0\,0 \ * \ * \ * \ * *, \quad f(h,0,1,1)=0 \ \wedge f(e,0,0,f)=0 \\
\Rightarrow \ & X^2 = * \ * \ * \,0\,0\,0 \ * \ * \ * \ *, \quad f(0,0,c,d)=0 \ \wedge f(e,0,0,f)=0 \ \wedge f(i,j,0,0)=0 \\
\Rightarrow \ & X^3 = * \ * \ * \,0\,0\,0\,0 \ * \ * *, \quad f(0,0,c,d)=0 \ \wedge f(e,0,0,f)=0 \ \wedge f(i,j,0,0)=0
\end{aligned}$$

   Then 0 is increasing, therefore the CA converges to $\overrightarrow{0}$.

   Let $X^0 \notin \{\overrightarrow{0}, \overrightarrow{1}, 01^{\frac{n}{2}}\}$ of wide lattice size n, i.e. X^0 contains at least yxx with $y, x \in \{0,1\}$, $x \neq y$, then the CA converges to $\overrightarrow{0}$ by the above proof.

2. If $X^0 = 01^{\frac{n}{2}}$ with n even, this is

$$\begin{aligned}
& X^0 = 010101010101 \cdots \\
\Rightarrow \ & X^1 = 111111111111 \cdots
\end{aligned}$$

   Thus the CA converges to $\overrightarrow{1}$.

Remark: The rules that satisfies the proposition 3.9 are: 33824, 50208,42016, 58400, 37920, 54304,46112, 62496.

Proposition 3.10

If $f(a,b,0,c)=0 \wedge f(d,0,e,f)=0\ \forall\ a,b,c,d,e,f \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the proposition 3.10 are: 32768, 49152, 32896, 49280, 32832, 49216, 32960, 49344.

Proposition 3.11

If $f(a,0,b,c)=0 \wedge f(d,e,1,0)=0 \wedge f(0,f,0,g)=0\ \forall\ a,b,c,d,e,f,g \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the proposition 3.11 are: 32768, 40960, 36864, 45056, 32896, 41088, 36992, 45184.

Proposition 3.12

If $f(0,0,a,b)=0 \wedge f(d,e,1,0)=0 \wedge f(f,g,0,1)=0 \wedge f(g,0,i,0)=0 \wedge f(0,j,0,k)=0 \wedge f(l,0,0,m)=0\ \forall\ a,b,c,d,e,f,g,h,i,j,k,l,m \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the proposition 3.12 are: 32768, 36864, 34816, 38912, 32896, 36992, 34944, 39040.

Proposition 3.13

If $f(a,0,0,b)=0 \wedge f(1,0,c,d)=0 \wedge f(0,1,e,f)=0 \wedge f(0,g,0,h)=0 \wedge f(i,0,j,0)=0$ $\forall\ a,b,c,d,e,f,g \in \{0,1\}$ then any CA of lattice size n, $n \geq 4$ converges to $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the Proposition 3.13 are: 32768, 49152, 40960, 57344, 36864, 53248, 45056, 61440, 32776, 49160, 40968, 57352, 36872, 53256, 45064, 61448.

Proposition 3.14

If $f(a,0,0,b)=0 \wedge f(0,0,c,d)=0 \wedge f(e,1,1,0)=0 \wedge f(1,1,0,1)=0 \wedge f(0,1,0,0)=0$ $\forall\ a,b,c,d,e,f,g \in \{0,1\}$ then any CA of odd lattice size n, $n \geq 4$ converges to $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$. If n is odd then the CA converges to $\overrightarrow{0}$ for $X^0 \notin \{\overrightarrow{1}, 01^{\frac{n}{2}}\}$.

Remark: The rules that satisfies the Proposition 3.14 are: 32768, 36864, 34816, 38912, 32896, 36992, 34944, 39040, 33824, 37920, 35872, 39968, 33952, 38048, 36000, 40096.

Proposition 3.15

If $f(a,0,0,b)=0 \wedge f(c,d,1,0)=0 \wedge f(e,f,0,1)=0 \wedge f(0,g,0,h)=0$ $\forall\ a,b,c,d,e,f,g,h \in \{0,1\}$ then any CA of lattice size n, $n \geq 4$ converges to $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the Proposition 3.15 are: 32768, 36864, 34816, 38912, 32896, 36992,

34944, 39040, 32776, 36872, 34824, 38920, 32904, 37000, 34952, 39048.

Proposition 3.16

If $f(a,0,0,b)=0 \wedge f(0,1,c,d)=0 \wedge f(e,f,0,1)=0 \wedge f(0,g,0,h)=0 \wedge f(i,0,j,0)=0$ $\forall\ a,b,c,d,e,f,g,h,i,j \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the Proposition 3.16 are: 32768 49152 36864 53248 34816 51200 38912 55296 32776 49160 36872 53256 34824 51208 38920 55304.

Proposition 3.17

If $f(a,0,0,b)=0 \wedge f(0,1,1,c)=0 \wedge f(d,e,0,0)=0 \wedge f(0,f,g,0)=0 \wedge f(1,0,1,1)=0$ $\forall\ a,b,c,d,e,f,g,h,i,j \in \{0,1\}$ *then any CA of lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$.

Remark: The rules that satisfies the proposition 3.17 are: 32768 49152 40960 57344 33824 50208 42016 58400 32776 49160 40968 57352 33832 50216 42024 58408

Proposition 3.18

If $f(a,0,0,b)=0 \wedge f(c,d,0,0)=0 \wedge f(0,e,f,0)=0 \wedge f(1,1,0,g)=0 \wedge f(h,1,1,0)=0$ $\forall\ a,b,c,d,e,f,g,h \in \{0,1\}$ *then any CA of odd lattice size n,* $n \geq 4$ *converges to* $\overrightarrow{0}$, $\forall\ X^0 \neq \overrightarrow{1}$. *If n is even, then the CA converges to* $\overrightarrow{0}$, $X^0 \notin \{\overrightarrow{1}, 01^{\frac{n}{2}}\}$.

Remark: The rules that satisfies the proposition 3.18 are: 32768, 34816, 32896, 34944, 33824, 35872, 33952

36000, 32776, 34824, 32904, 34952, 33832, 35880, 33960, 36008.

Proposition 3.19

If $f(x_1, x_2, x_3, x_4) = x_1x_2\overline{x}_3x_4 + x_1\overline{x}_2x_3x_4 + x_3x_2$, this is $f(0,0,a,b) = 0 \wedge f(c,0,0,d) = 0 \wedge f(e,1,1,f) = 1 \wedge f(g,0,h,0) = 0 \wedge f(0,i,0,j) = 0$ (rule 59584) then any CA of lattice size n, $n \geq 4$ with X^0 containing $00 \vee 010$ converges to $\overrightarrow{0}$, otherwise the CA converges to $\overrightarrow{1}$.

Proof. Suppose that X^0 contains 00. Thus

$$\begin{aligned} & X^0 = *\,0\,0\,*\,*\,* \\ \Rightarrow\ & X^1 = *\,0\,0\,0\,*\,* \\ \Rightarrow\ & X^2 = *\,0\,0\,0\,0\,* \\ \Rightarrow\ & X^2 = *\,0\,0\,0\,0\,0 \end{aligned}$$

That is, 0 is increasing, therefore the CA converges to $\overrightarrow{0}$.
Suppose that X^0 contains 010. Thus

$$\begin{aligned} & X^0 = *\,0\,1\,0\,*\,* \\ \Rightarrow\ & X^1 = *\,*\,0\,0\,*\,* \end{aligned}$$

Here we are in the previous case, thus the CA converges to $\overrightarrow{0}$.
Suppose that X^0 does not contains the two 0s together (00) and it does not contains 1 isolated 010, then X^0 it contains expressions of the form 1111, 1110, 1101, 1011, 0111 , 0110 and $f(1,1,1,1) = f(1,1,1,0) = f(1,1,0,1) = f(1,0,1,1) = f(0,1,1,1) = f(0,1,1,0) = 1$, therefore, the CA conveges to $\overrightarrow{1}$.

Neighbor Zeros Classification Problem–NzCP

Neighbor Zeros Classification Problem–NzCP

$$NzCP: \begin{cases} S = S_{all} \\ \mathcal{P} = \{x \in S_{all} : x \text{ contains } 00 \vee 010\} \\ \mathcal{Q} \equiv S_{all} \setminus \mathcal{P} = \{x \in S_{all} : x \text{ does not contains } 00 \wedge 010\} \\ \mathcal{A}_{\mathcal{P}} = 0^* \cap S_{all} \text{ and } \mathcal{A}_{\mathcal{Q}} = 1^* \cap S_{all} \end{cases}$$

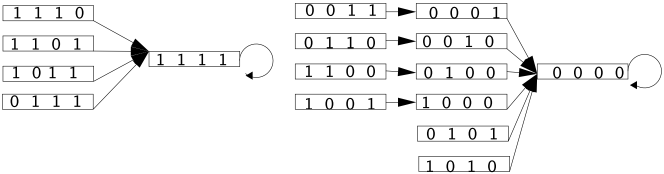
Remark:

By Proposition 3.19 we have the NzCP has a solution for all CA radius $r = 1.5$.

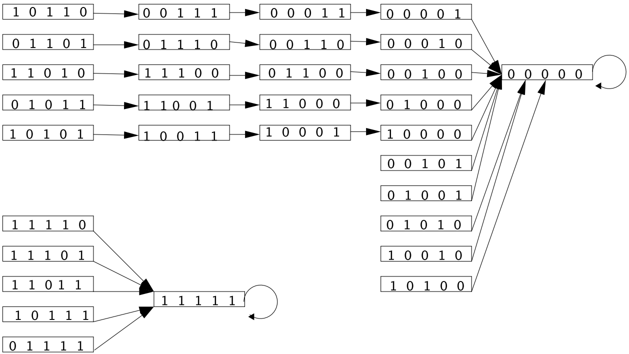
Remark:

The NzCP verifies the closing property.

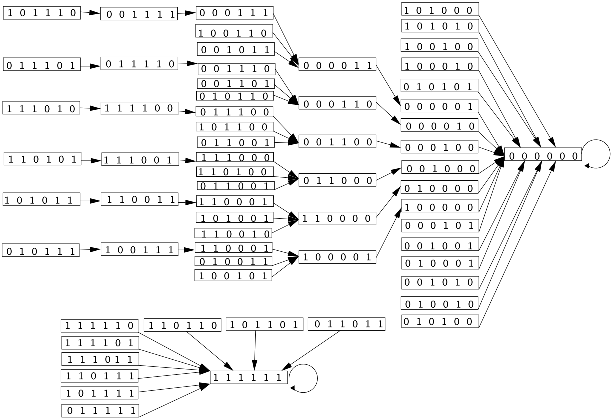
Rule 59584
CA of lattice size 4



Rule 59584
CA of lattice size 5



Rule 59584
CA of lattice size 6



Proposition 3.20

If $f(x_1, x_2, x_3, x_4) = x_3x_4 + x_2x_3\overline{x}_4 + x_1x_2\overline{x}_3$, *that is* $f(1,1,a,b) = 1 \wedge f(c,1,1,d) = 1 \wedge f(e,f,1,1) = 1 \wedge f(g,0,0,h) = 0$ *(rule 63688) then any CA of lattice size n,* $n \geq 4$ *with* X^0 *containing* 11 *converges to* $\overrightarrow{1}$*. Conversely, if* X^0 *does not contains* 11 *the CA converges to* $\overrightarrow{0}$*.*

Proof. Suppose that X^0 contains 11 then

$$\begin{aligned} X^0 &= * \ * \ *1\ 1\ * \ * \\ \Rightarrow X^1 &= * \ * \ *1\ 1\ 1\ * \\ \Rightarrow X^2 &= * \ * \ *1\ 1\ 1\ 1 \end{aligned}$$

Then in this case 1 is increased, therefore the CA converges to $\overrightarrow{1}$.

Suppose that X^0 does not contains 11 then X^0 It contains only expressions of the form 0000, 1001, 1010, 0101, 1000, 0001 and $f(0,0,0,0) = f(1,0,0,1) = f(1,0,1,0) = f(0,1,0,1) = f(1,0,0,0) = f(0,0,1,0) = f(0,1,0,0) = f(0,0,0,1) = 0$, Therefore $X^1 = \overrightarrow{0}$.

Clustered 1s Classification Problem–C1CP

Clustered 1s Classification Problem–C1CP

$$C1CP: \begin{cases} S = S_{all} \\ \mathcal{P} = \{x \in S_{all} : x \text{ contains } 11\} \\ \mathcal{Q} \equiv S_{all} \backslash \mathcal{P} = \{x \in S_{all} : x \text{ does not contains } 11\} \\ \mathcal{A}_{\mathcal{P}} = 1^* \cap S_{all} \text{ and } \mathcal{A}_{\mathcal{Q}} = 0^* \cap S_{all} \end{cases}$$

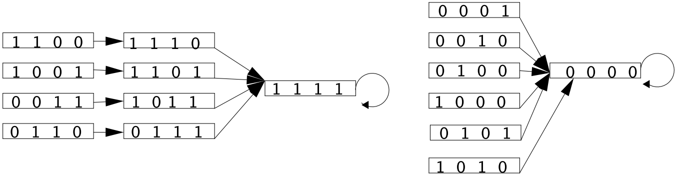
Remark:

By Proposition 3.20 we have the C1CP has a solution for all CA radius $r = 1.5$.

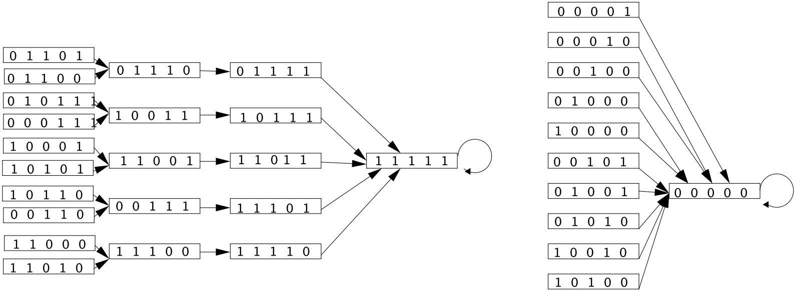
Remark:

The C1CP verifies the closing property.

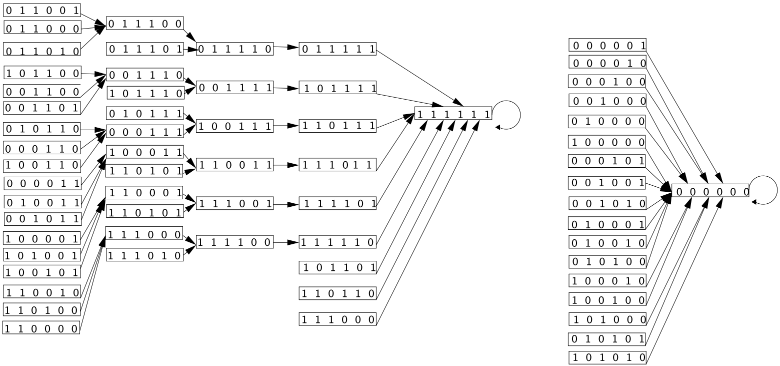
Rule 63688
CA of lattice size 4



Rule 63688
CA of lattice size 5



Rule 63688
CA of lattice size 6



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Recognisable Languages

There were four recognisable languages identified by this CA ($r = 1.5$)

1. $\{0,1\}^*0 \longrightarrow \overrightarrow{0}$
2. $\{0,1\}^*\{010,00\} \longrightarrow \overrightarrow{0}$
3. $\{0,1\}^*\{11\} \longrightarrow \overrightarrow{1}$
4. $\begin{cases} \{0,1\}^*0 \longrightarrow \overrightarrow{0}, & \text{n odd} \\ 01^{\frac{n}{2}}1^* \longrightarrow \overrightarrow{1}, & \text{n even} \end{cases}$

Work in Progress

-I am searching for classification problems of CA of radius 2.

Future work

-Analyse and characterize classification problems.

-I hope obtains: well defined global classification problem that is solvable or find a well defined classification problem and show that it has no solution.

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References

- Lee, K. M., Xu, H., and Chau, H. F. (2001). Parity problem with a cellular automaton solution. *Physical Review E*, 64(2), 026702.
- Chopard, B. (2012). Cellular automata modeling of physical systems. *Computational Complexity: Theory, Techniques, and Applications*, 407-433.
- Betel, H., de Oliveira, P. P., and Flocchini, P. (2012). On the parity problem in one-dimensional cellular automata. *arXiv preprint arXiv:1208.2758.*
- Ermentrout, G. B., and Edelstein-Keshet, L. (1993). Cellular automata approaches to biological modeling.*Journal of theoretical Biology*, 160(1), 97-133.
- Eric Goles, Marco Montalva-Medel and Pedro P.B. de Oliveira Unsolvability of classification problems over cyclic binary configurations, by one-dimensional cellular automata
- Alber, M. S., Kiskowski, M. A., Glazier, J. A., and Jiang, Y. (2003). On cellular automaton approaches to modeling biological cells. In Mathematical Systems Theory in Biology, Communications, Computation, and Finance (pp. 1-39). *Springer New York.*
- Fuks-acute, H. (1997). Solution of the density classification problem with two cellular automata rules. *Physical Review E,* 55(3), R2081.
- Busic, A., Fates, N., Mairesse, J.,and Marcovici, I. (2012). Density classification on infinite lattices and trees. In LATIN 2012: *Theoretical Informatics* (pp. 109-120). Springer Berlin Heidelberg.
- Land, M., and Belew, R. K. (1995). No perfect two-state cellular automata for density classification exists. *Physical review letters*, 74(25), 5148.
- Wolfram, S. (1986). Tables of cellular automaton properties. *Theory and Applications of Cellular Automata*, 485-557.