

COMPLEJIDAD COMPUTACIONAL DEL AUTOMATA DE MAYORÍA CON SIGNOS

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Desafíos matemáticos e informáticos para la
construcción y análisis de redes de regulación
biológica

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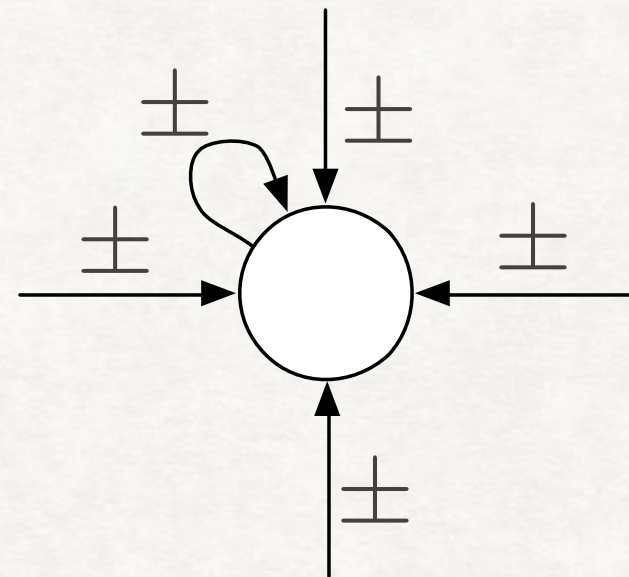
MAYORÍA BI-DIMENSIONAL CON SIGNOS

$$x \in \{-1, 1\}^{n^2}$$

$$N[v] = \{v, v_n, v_s, v_e, v_w\}$$

(vecindad von Neuman)

$$A = (a_{u,v})_{u,v \in \mathbb{Z}^2}, a_{u,v} \in \{-1, 1\}$$



$$(F_A(x))_v = \begin{cases} 1 & \text{if } \sum_{u \in N(v)} a_{u,v} x_u > 0, \\ -1 & \text{otherwise} \end{cases}$$

COMPLEJIDAD COMPUTACIONAL

$$A = (a_{u,v})_{u,v \in \mathbb{Z}^2}, a_{u,v} \in \{-1, 1\}$$

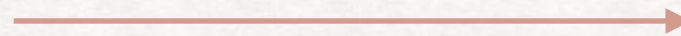
Prediction(A)

Input: A periodic configuration x and a site v .

Question: Does there exists $T > 0$ such that $(F_A^T(x))_v = 1$

"Capacidad de simulación
de circuitos"

Circuitos monótonos
planos



Circuitos planos



Circuitos monótonos
planos re-utilizables

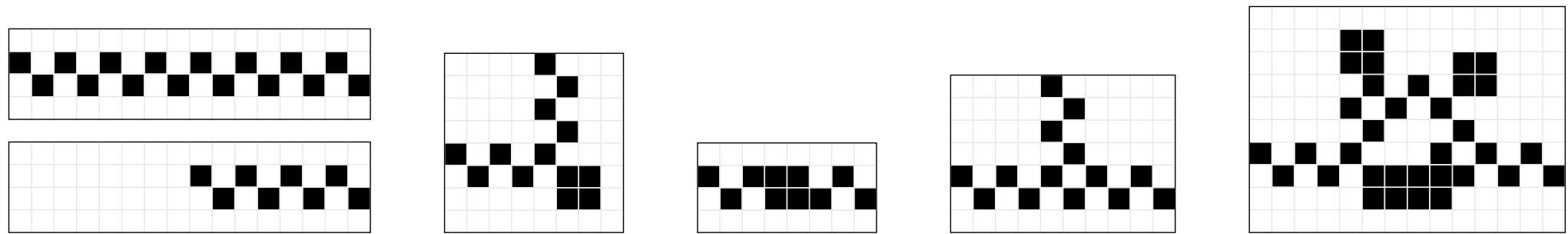


Cota inferior de la
complejidad

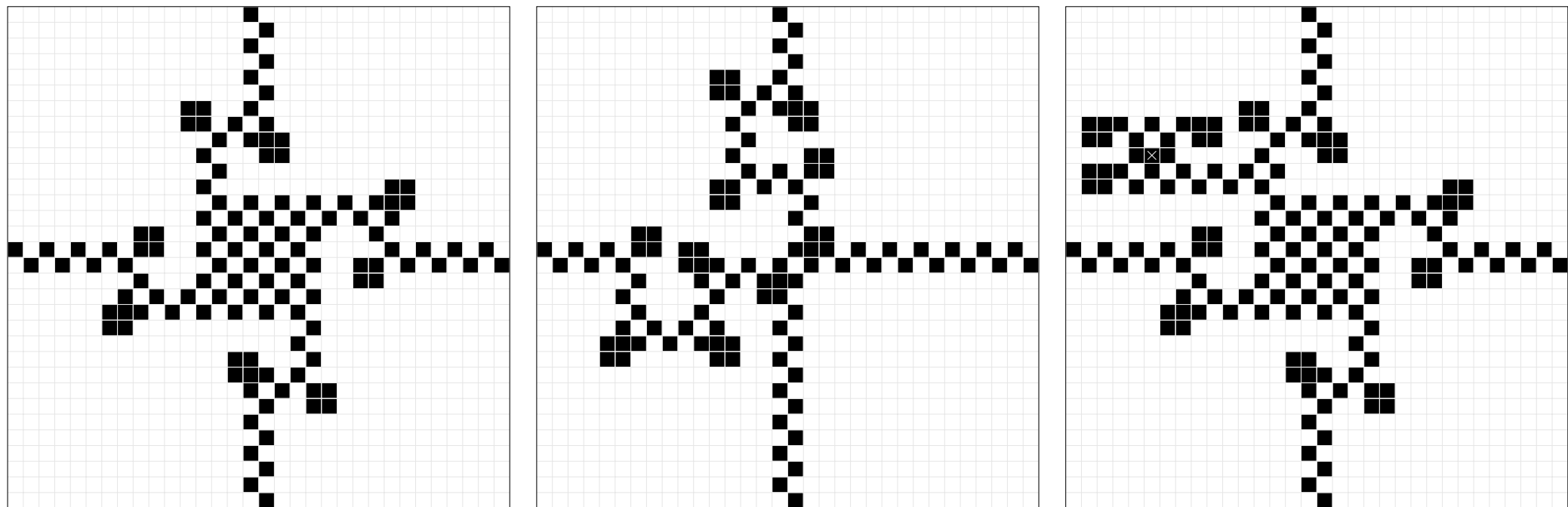
NC¹-Hard

P-Hard
(Turing-Universal)

PSPACE-Hard
(Intrínsecamente-Universal)



(a) A wire is constantly blinking (left top) and transmits a signal by being consumed (left bottom). A turn (center left). An obstructor prevents a signal (center). A multiplier (center right). These gadgets can straightforwardly be composed to simulate a wiring toolkit W . Plus a diode (right).

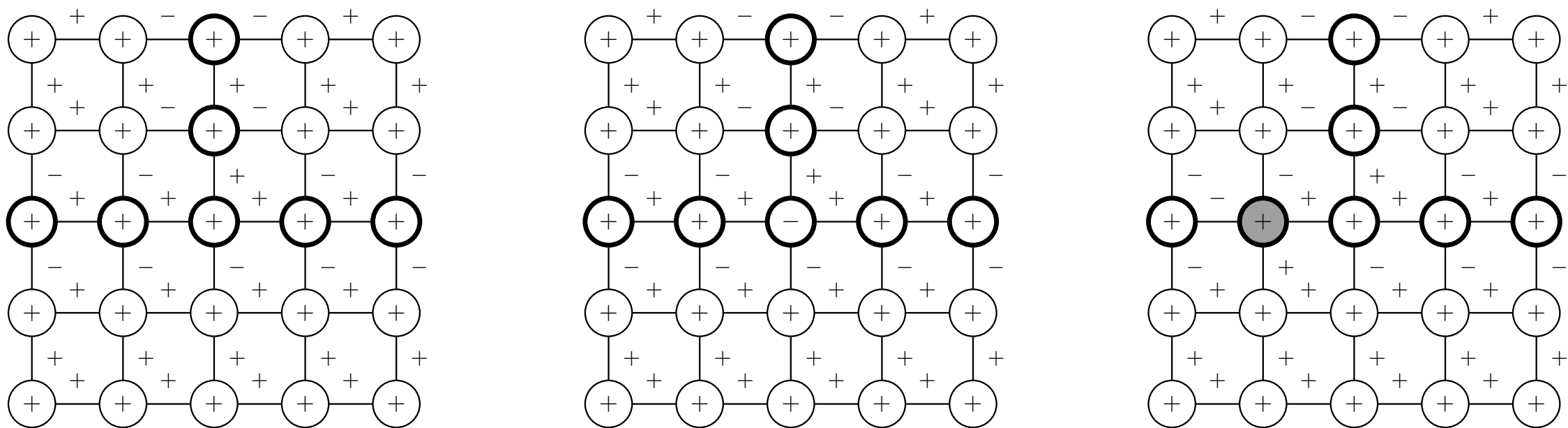


(b) OR (left), AND (center) and FUSIBLE (right, with a distinguished cell) gates. These gates have a particular feature: the four sides are undifferentiated inputs/outputs. For example, the AND gates waits for any two signals to arrive, and then sends a signal to the two remaining sides. One can easily transform them into classical gates using diodes.

Mayoría está en P y es NC^1 -Hard

CASO NO UNIFORME (MOTIVACIÓN?)

Existe una matriz A simétrica (i.e $a_{u,v} = a_{v,u}$),
tal que **Prediction**(F_A) es P-Completo (Turing-Universal)

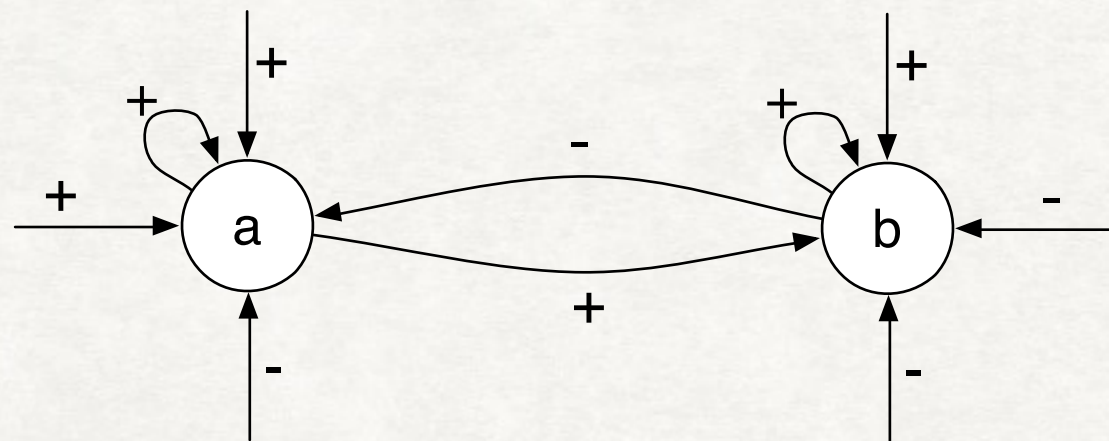
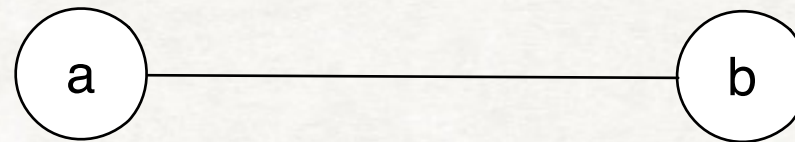


(b) AND gate, OR gate and $(\neg west)AND(north)$ gadgets. Inputs are from north and west, and outputs to east.

Existe una matriz A para la cual **Prediction**(F_A) es
PSPACE-Completo (Intrínsecamente-Universal)

SI ES UN AUTOMATA...

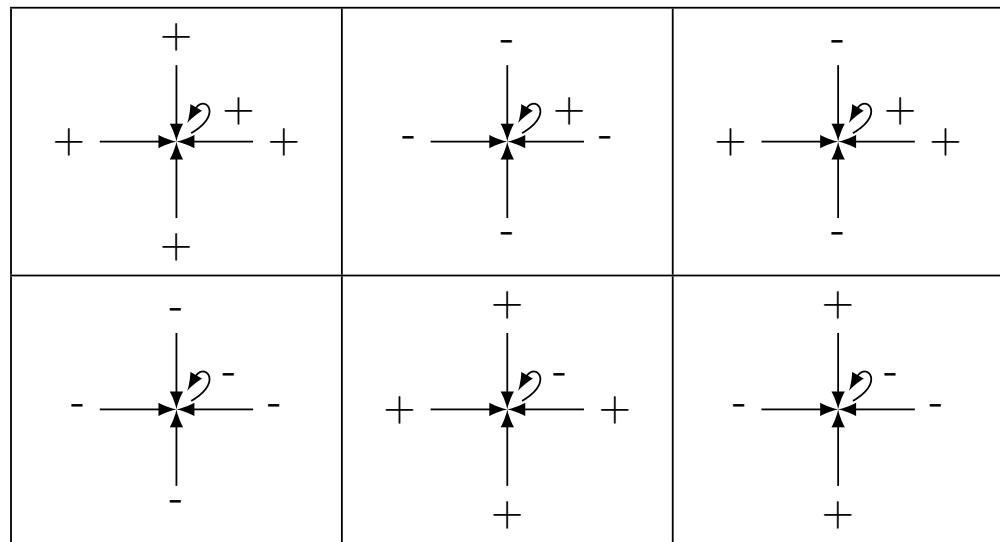
$$A \rightarrow W_A = (C, N, S, E, O) \in \{-1, 1\}^5$$



$$W_A = (1, 1, -1, -1, 1)$$

SI ES UN AUTOMATA...

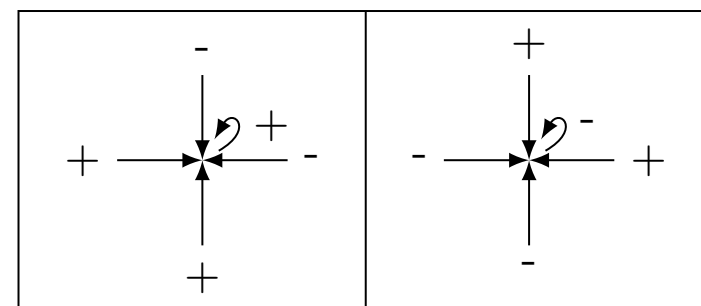
Hay 32 reglas, que se reducen a 12 salvo rotación.



Simétricas

NC¹-Hard, en P,

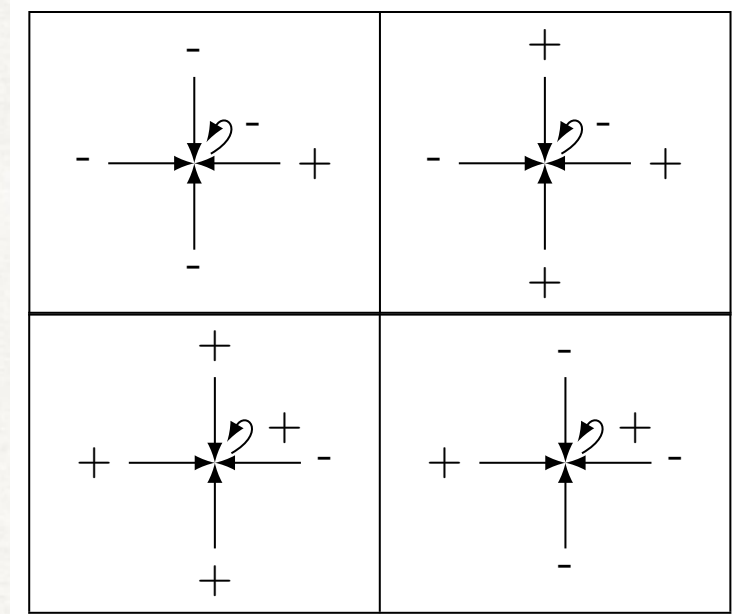
Ciclos de largo a lo más 2



Anti-simétricas

? (en PSPACE)

Puede tener ciclos
de largo n^2



Asimétricas

P-Hard, en PSPACE,

Puede tener ciclos
super-poly

Y SI NO ES MAYORÍA...

Las equivalencias entre casos uniformes simétricos, antisimétricos y asimétricos son válidas.

Caso "AND-NOT":

$$(F_A(x))_v = \begin{cases} 1 & \text{if } \sum_{u \in N(v)} a_{u,v} x_u = 5 \\ -1 & \text{otherwise} \end{cases}$$

Existe una matriz A antisimétrica (no uniforme) para la cual
Prediction(F_A) es PSPACE-Completo
(Intrínsecamente-Universal)