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A Comparison of Inferences about Containers and Surfaces in Small-Scale and Large-Scale Spaces*

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Abstract

Inference mechanisms about spatial relations constitute an important aspect of spatial reasoning as they allow users to derive unknown spatial information from a set of known spatial relations. When formalized in the form of algebras, spatial-relation inferences represent a mathematically sound definition of the behavior of spatial relations, which can be used to specify constraints in spatial query languages. Current spatial query languages utilize spatial concepts that are derived primarily from geometric principles, which do not necessarily match with the concepts people use when they reason and communicate about spatial relations. This paper presents an alternative approach to spatial reasoning by starting with a small set of spatial operators that are derived from concepts closely related to human cognition. This cognitive foundation comes from the behavior of image schemata, which are cognitive structures for organizing people's experiences and comprehension. From the operations and spatial relations of a small-scale space, a container-surface algebra is defined with nine basic spatial operators—*inside, outside, on, off*, their respective converse relations—*contains, excludes, supports, separated_from,* and the identity relation *equal*. The container-surface algebra was applied to spaces with objects of different sizes and its inferences were assessed through human-subject experiments. Discrepancies between the container-surface algebra and the human-subject testing appear for combinations of spatial relations that result in more than one possible inference depending on the relative size of objects. For configurations with small-scale and large-scale objects larger discrepancies were found because people use relations such as *part of* and *at* in lieu of *in*. Basic concepts such as containers and surfaces seem to be a promising approach to define and derive inferences among spatial relations that are close to human reasoning.

1. Introduction

Users of geographic information systems (GISs) and spatial databases typically formulate spatial queries whose constraints are based on spatial relations (Roussopoulos & Leifker 1985, Egenhofer & Frank 1988, Güting & Schneider 1995). Examples are such requests as "find the largest town *in* Penobscot county" and "find all National Parks that are located *on* an island." Models for spatial relations in GISs have been traditionally derived from geometric properties and often limited to computations within a Cartesian coordinate space. This approach, however, constrains the

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representation of spatial information, because it assumes the existence of complete coordinate tuples for each spatial entity. Moreover, geometric properties may not necessarily capture all information that people use to reason about space. To complement traditional spatial data models, qualitative representations of space have been investigated (Hernández 1994). Within the realm of qualitative spatial reasoning, the study of algebras for spatial relations has been of growing interest in the GIS community (Smith & Park 1992, Egenhofer & Sharma 1993). These algebras focus on the inferences made from compositions of spatial relations, such as topological relations (Randell *et al.* 1992, Egenhofer & Franzosa 1991, Egenhofer 1994), cardinal directions (Frank 1991, Freksa 1991, Papadias & Sellis 1994), approximate distances (Hernández *et al.* 1995, Hong *et al.* 1995), and their combinations (Nabil *et al.* 1995, Sharma & Flewelling 1995). While these formalizations are mathematically sound, there has been little concern as to whether existing spatial-relation algebras are cognitively plausible (Hirtle 1991, Hernández 1994).

Naive Geography (Egenhofer & Mark 1995) promotes an alternative path to modeling space through the use of concepts that are grounded in human experiences and, therefore, are expected to match more closely with human thinking. The consideration of such commonsense concepts may lead to new theories for developing GISs with user interfaces and spatial reasoning capabilities that respond to their users' expectations. In search for alternatives, researchers in geographic information science have been inspired by cognitive (Lakoff 1987) and linguistic (Talmy 1983) theories about space as a foundation for easier-to-use GISs (Mark & Frank 1991). In cognitive science, the concept of *image schemata* (Lakoff & Johnson 1980, Johnson 1987) emphasizes the experiential dimension of human thinking. Image schemata are recurrent patterns, such as *container*, *surface*, *link*, and *path*, that can be extended and metaphorically projected to make new experiences meaningful. They aim at representing what may be common in the way people understand and think about their perceptions of the world. Although image schemata are more abstract concepts than visual representations, they are idealized conceptual models of human perception and cognition (Mark 1989) and, therefore, they are basis for the development of user interfaces and visual languages that are easy to learn and employ by many users, particularly users from different disciplines, cultures, and linguistic groups (Mark *et al.* 1989).

Since image schemata may provide the structures for understanding the meaning of spatial relations (Mark 1989, Freundschuh & Sharma 1996), we use them here as a foundation for constructing an intuitive and cognitively-plausible *spatial-relation algebra*. In the theory of image schemata, all people learn the same basic spatial concepts through essentially the same bodily experiences and, therefore, can share the same knowledge without a need for detailed explanations or instructions. Image schemata are expected to serve as a basis for future query languages and inference mechanisms that reflect basic human reasoning. For example, a scenario in which merchandise on a platform must be transported via the interstate to a warehouse can be described at a higher level of abstraction through the combination of the image schemata involved, given the mappings that the merchandise are *objects*, the platform is a *surface*, the interstate is a *path*, and the warehouse is a *container*. Many image schemata are fundamentally spatial in nature (Mark 1989). The spatial concepts underlying image schemata, however, are not necessarily limited to spatial applications since image schemata are commonly used through metaphorical projections in non-spatial domains; therefore, query languages developed from image schemata can be widely applicable beyond spatial applications.

Instead of analyzing users' behaviors in using a given set of GIS operations, we focus on a small set of operations that characterize the behaviors of two major image schemata: the *container* and the *surface*. Container and surface were found to be the two most basic image schemata in a complexity ranking derived from the spatial concepts children learn (Freundschuh & Sharma 1996). By using the behavior of image schemata, this approach follows the notion of use-based semantics (Kuhn 1994), which better captures the meaning of objects in terms of the operations people perform with them than do descriptions of attributes. Of particular interest are those inferences that can be made from combinations of image schemata. In algebraic terms, such inferences are referred to as the composition of relations. Composition combines two binary relations over a common object to determine the relation(s) between the linked objects. A special case of composition occurs if the same relations are composed, which characterizes a transitive relation.

Taking as a case study a room space, spatial relations associated with the behavior of containers and surfaces were derived and specified in terms of a *relation algebra* (Tarski 1941). The influence of scale on the sensibility of this algebra is addressed by applying and comparing the spatial inferences derived from the room space with similar inferences in a geographic space. A human-subject testing was performed to evaluate if people would support or reject inferences made by our algebra in small-scale and large-scale spaces.

The remainder of this paper is structured as follows: Section 2 describes the main characteristics of image schemata. Section 3 presents the container-surface algebra for a small-scale space. This container-surface algebra is then applied to configurations with large-scale scale and with combinations of small- and large-scale objects (Section 4). Description and analysis of a human-subject testing are given in Section 5. Conclusions and future work are presented in Section 6.

2. Image Schemata

Lakoff and Johnson (1980) defined *image schemata* as recurrent patterns that people learn through physical repetitive experiences. For example, infants experience the image schema of a *container* when they put food into their mouths. Once people have developed an image schema, they extend it, transform it, and metaphorically project it to produce meaningful situations. Johnson (1987) pointed out that "image schemata are pervasive, well defined, and full of sufficient internal structure to constrain our understanding and reasoning." Image schemata are composed of parts and relations that allow for an organization of many different perceptions or events. Image schemata imply that experiences are organized into meaningful structures before and independently of concepts; however, concepts can impose more constraints to already existing structures. Contrary to concepts, image schemata are dynamic structures operating at a higher level of abstraction and generality than concrete images. Thus, they are adaptable structures according to the context for organizing situations; however, they become relatively stable by being located in our network of meaning.

Johnson (1987) presented a partial list of image schemata (Table 1), which cover only what he considers to be the most important image schemata. The image schemata presented there are described informally in natural language and lack the formal rigor in order to make computational inferences about them or to incorporate them into query languages. With respect to spatial reasoning, important characteristics of image schemata are their limited number and capacity for constraining inferences. Their metaphorical projections to concepts in the real world are the basis for meaningful relations.

Container	Balance	Full-Empty	Iteration	Compulsion
Blockage	Counterforce	Process	Surface	Restraint Removal
Enablement	Attraction	Matching	Part-Whole	Mass-Count
Path	Link	Collection	Contact	Center-Periphery
Cycle	Splitting	Merging	Object	Scale
Superimposition				

Table 1: The partial list of image schemata by Johnson (1987).

Many image schemata are associated with spatial relations. For example, given the image schema *container*, whose structural elements are the interior, boundary, and exterior, the *in-out* orientation becomes the differentiation, separation, and enclosure between interior and exterior. People may experience an image schema in different ways since they may project this schema onto objects of different sizes, nature, and types. For instance, objects associated with a container in a small-scale or table-top space (Zubin 1989), such as a box, cup, or drawer, are experienced by inserting objects into them. In a large-scale space (Kuipers 1978), on the other hand, a country may also be considered a container as people experience it by traveling and crossing its borders. In both small- and large-scale spaces, the same basic behavior (containment) and structural elements (interior, boundary, and exterior) are present.

Image schemata may coexist if they are present at the same time, but their meanings do not depend on each other. A stronger notion is if they are related, i.e., the meaning of one image schema depends on the presence of another image schema. A physical *container*, for example, is characterized by virtue of its capacity to have objects inside. Thus, it is constrained by the space available to contain objects, which refers to the *full-empty* schema. Moreover, the container's boundary may be considered as the periphery with respect to the center of the container, which is described by the *center-periphery* schema. Finally, the *center-periphery* schema relates to the *near-far* schema as our perception extends from the center (i.e., near) to the periphery (i.e., far). A *surface* is characterized by the capacity to put on or take off objects. The ideal image of a surface assumes that it is possible to put an object on the surface when there is a contiguity property—*contact* schema—between the surface and the object and when the surface can *support* the object. Surface also has a capacity (area) associated with the contiguity property to put objects on. This is similar to the case of the space that is available to insert objects into a container. In this sense, a surface is full if its area (capacity of contiguity) is completely covered by objects on it.

The image schema adopted to describe the behavior of an object determines the possible spatial relations between this object and other objects within a configuration. For instance, if a desk acts as a surface onto which we can put a book (Figure 1a), the spatial relation between the desk and the book is *on*. On the other hand, the same desk can be used as a container into which we can move a book (Figure 1b) such that the spatial relation between the desk and the book is *in*. In the first scenario a person could touch the book without manipulating the desk first. In the second case, the book is completely surrounded by the desk and, therefore, a person cannot access the book unless the desk is opened. Thus, the spatial relation between the book and the desk depends on what image schemata (container or surface) describes the behavior of the desk.

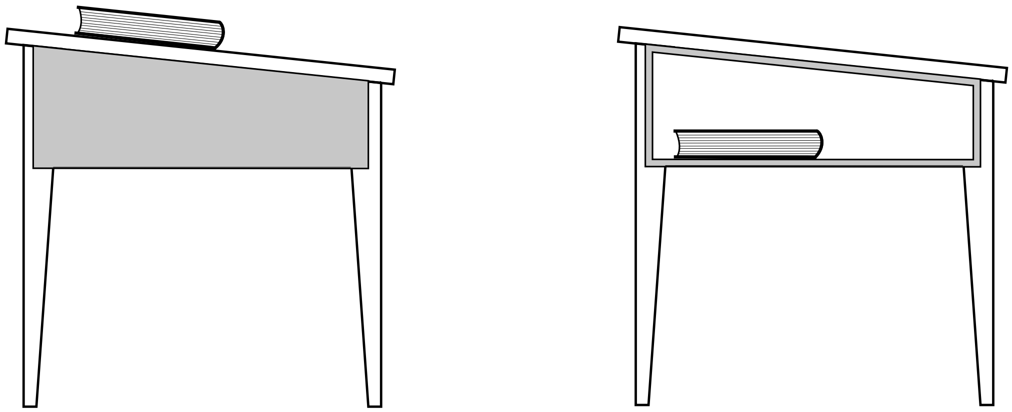


Figure 1: Spatial relations between a desk and a book: (a) the book on the desk and (b) the book in the desk.

A series of recent investigations shares the interest in image schemata in spatial reasoning. Mark and Frank (1996), for instance, used image schemata to describe the experiential and formal models of geographic scenes. Likewise, image schemata were used to structure space for wayfinding tasks in airports (Raubal *et al.* 1997). Image schemata may explain how different spatial relations are used in natural language through prepositions (Mark 1989), because they are conceptual models of human perception and cognition; therefore, they form an excellent basis for describing spatial scenes and for designing GIS query languages. Mark (1989) went further in the use of image schemata by suggesting that image schemata should be used to define good user interfaces. Thus, user interfaces including commands and query languages would be compatible with the views that users have of the system. Similarly, Kuhn and Frank (1991) and later again Kuhn (1993), supported the idea that image schemata and their metaphorical mappings are the fundamental theories to build efficient user interfaces. Freundschuh and Sharma (1996) linked image schemata with the spatial concepts that children learn through story books and suggested that there is a progressive process of spatial knowledge understanding and that some image schemata seem to be more fundamental or basic than others. They also presented how locative prepositions relate to image schemata stating, for example, while *in* and *out* correspond to locative prepositions associated with a *container*, *on* and *off* correspond to locative prepositions associated with a *surface*.

3. Inferences about Containers and Surfaces in a Small-Scale Space

We pursue a top-down approach to spatial reasoning through a study of a prototypical case with semantically meaningful objects and their operations, rather than attempting to derive spatial relations for geometric parts (points, lines, polygons) from a Cartesian coordinate space. The basis for the formalization of the container-surface algebra is an analysis of a room space, a concrete scenario in which people interact with spatial objects through bodily experiences (Rodríguez & Egenhofer 1997, Egenhofer & Rodríguez in press). A room is a ubiquitous case of a small-scale space where people manipulate objects and experience objects from one standpoint (Kuipers 1978, Zubin 1989). As such, the room space constitutes a representative scenario in which people experience recurrent manifestations of image schemata.

This study considers a neatly organized room space with six major objects: a box, a ball, a table, a sheet of paper, a pen, and a room itself. These objects embody the image schemata *container*—e.g., the table *in* the room or the ball *in* the box—and *surface*—e.g., the pen *on* the paper or the paper *on* the table—and allow for inferences about their associated spatial relations—e.g., the pen *on* the table and the table *in* the room implies that the pen is *in* the room as well. We assume, for the time being, that all objects may only be completely on or off a surface, i.e., no part of, say, the paper may extend beyond the tabletop. Similarly, all objects can be only completely in or out of a container. The discussion of partially on and off and partially inside and outside relates to the *part-whole* image schema, which would introduce more complex variations of the simpler but more fundamental cases considered here. The ontology of the room space can be generalized to six configurations involving containers and surfaces (an object is an item that is neither a container nor a surface):

- An object *inside* a container.
- An object *on* a surface.
- A container (with an object *inside*) *in* another container.
- A surface (with an object *on* top) *on* another surface.
- A container (with an object *inside*) *on* a surface.
- A surface (with an object *on* top) *in* a container.

3.1 Primitive Relations of Containers and Surfaces

From the analysis of the room space, the definition of spatial relations follows the formalism described by relation algebras. Defining spatial relations as part of a relation algebra allows us to use relations as variables and properties of this algebra as mechanisms of inferences. To define a relation algebra, a set of primitive relations must be defined. This set of primitive relations must contain pairs of converse relations, complement relations, an identity relation, and an empty relation.

The analysis of the room space distinguishes different spatial relations for the container and surface schemata. The basic spatial relation for the container is *inside*, which results from the operation of moving a (smaller) object into a (larger) object that plays the role of a container. The converse relation to *inside* is *contains*—if an object A is *inside* a container B then B *contains* A, and vice-versa. The relations *inside* and *contains* have their respective negations, called *outside* and *excludes,* if we consider two different objects. If A is different to B and A is not *inside* the container B, then A must be *outside* of B. Likewise, if the container B does not *contains* A, then B *excludes* A. To enable a complete set of inferences about containers, the identity relation $equal_C$ is introduced, which holds only between a container and itself or between an object and itself. Since converse and negation are associative operators for spatial relations between two different objects, *excludes* can be defined in two ways: (1) as the negation of the converse of *inside* and (2) as the converse of the negation of *inside*.

The set of the five primitive container-relations forms the universal relation, denoted by $\mathcal{U}_C$. The elements of the universal relation provide a complete coverage (i.e., any binary configuration with a container is described by one of the five relations) and they are mutually exclusive (Expression 1). For each relation there exists the complement, which is the universal relation minus that relation. A particular role plays the complement to $equal_c$, as it establishes the diversity relation.

$$\begin{aligned} &A\ \textit{is a container} \text{ or } B\ \textit{is container} \Leftrightarrow A\ \textit{inside}\ B \text{ xor } A\ \textit{contains}\ B \text{ xor} \\ &\qquad A\ \textit{outside}\ B \text{ xor } A\ \textit{excludes}\ B \text{ xor} \\ &\qquad A\ \textit{equal}_C\ B \end{aligned} \tag{1}$$

The relations for a surface are defined similarly. The prototyping relation for a surface is *on*, which results from moving an object onto a surface. The negation of *on* is *off*, and the converse to *on* and *off* are called *supports* and *separated_from* respectively. The relation $equal_S$ forms the identity relation between two surfaces or objects, and the five relations *on*, *off*, *supports*, *separated_from*, and $equal_S$ form the universal relation for surfaces, $\mathcal{U}_S$.

In the container-surface algebra, both types of relations occur simultaneously, which means that there exists a universal relation as the integration of the universal relation of containers and the universal relation of surfaces. The definitions and properties of the individual relations stay unchanged, however, only one single identity relation exists in this set of combined container-surface relations, denoted by $equal_{C\&S}$. The complement of a relation is based on its definition, i.e., the universal relation minus that particular relation. So while the complement of the relation *inside* in the container algebra consists of the four relations *outside*, *contains*, *excludes*, and $equal_C$, it encompasses in the combined container-surface algebra the eight relations *outside*, *contains*, *excludes*, *on*, *off*, *supports*, *separated_from*, and $equal_{C\&S}$. The nine basic relations of the container-surface algebra are depicted graphically through a set of icons that expose prototypical cases (Table 2).

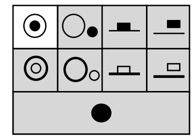
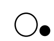
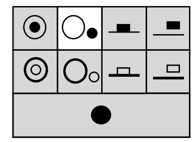
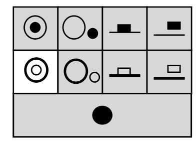
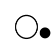
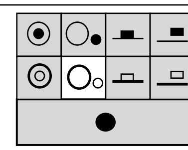
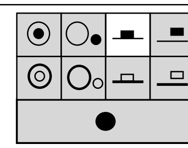
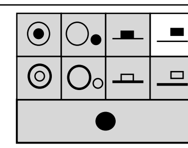
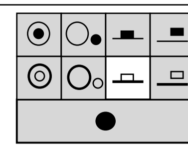
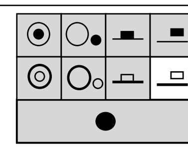
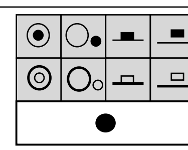
Image Schema	Relation	Result of operation	Icon for relation	Converse relation	Complement
container	A *inside* B	A moved into container B			
container	A *outside* B	A removed from container B			
container	B *contains* A	A moved into container B			
container	B *excludes* A	A removed from container B			
surface	A *on* B	A moved onto surface B			
surface	A *off* B	A removed from surface B			
surface	B *supports* A	A moved onto surface B			
surface	B *separated_from* A	A removed from surface B			
container or surface	A $equal_{C\&S}$ B				

Table 2: Basic relations of the container-surface algebra (dark boxes denote complement relations) (Egenhofer & Rodríguez in press).

3.2 Compositions

Much of the semantics of these spatial relations is captured by the composition operation, also called the relative product, of a relation algebra. Composition combines two relations over a common argument to determine the relation between the linked arguments (Expression 2). For example, the composition of *inside* with *outside* implies *outside* (because if A is *inside* of B and B is *outside* of C, then A must be *outside* of C as well). Composition may result in more than one possible relation (e.g., the composition *off* with *on* results in *on* xor *off*). If the composition imposes no constraints, then it results in the universal relation. On the other end of the scale, if the composition is impossible, then it results in the empty relation.

$$x;y = \{< a,c >: \exists_b < a,b > \in x \ \wedge \ < b,c > \in y\} \qquad (2)$$

The composition of spatial relations was derived from separate container and surface algebras, which got integrated into a single container-surface algebra (Table 3). Given the composition table, the necessary properties of a relation algebra (Tarski 1941) can be assessed. Properties of a relation algebra were examined for the container-surface algebra by using a program written in C++. Using the set-theoretic operations, i.e., union (+), intersection (•), and complement (–), and considering the binary operator corresponding to composition (;), the unary operator corresponding to converse (^{c}), the universal relation $U_{C\&S}$, and the identify relation $equal_{C\&S}$, the seven properties of the container-surface algebra are:

- Each composition with the identity relation is idempotent (i.e., $x;equal_{C\&S} = x$).
- The composition with a union of relations is equal to the union of the compositions with each of the elements of the union (i.e., $(x+y);z \ = \ x;z+y;z$).
- The converse of a converse relation is equal to the original relation (i.e., $(x^c)^c = x$).
- The converse of a union of relations is equal to the union of the converse relations of each of the elements of that union (i.e., $(x+y)^c = x^c + y^c$).
- The converse relation of a composition is equal to the composition of the converses of the two relations, taken in reverse order (i.e., $(x;y)^c = y^c;x^c$).
- A variation of De Morgan's Theorem K (i.e., $x^c;-(x;y)+-y = y$) holds.
- The associative property of the composition (i.e., $(x;y);z \ = \ x;(y;z)$) that applies to the container and surface algebras is no longer applicable for the container-surface algebra. Although the container-surface algebra is not associative, it is a semiassociative relation algebra (Maddux 1982, Andréka *et al.* 1988), because it satisfies $x;U_{c\&s};U_{c\&s} = (x;U_{c\&s});U_{c\&s} = x;(U_{c\&s};U_{c\&s}) = x;U_{c\&s}$.

Since the container-surface algebra is semiassociative, it is possible to draw different conclusions from two different reasoning paths, where one of them produces a subset of the possible spatial relations derived from the other one. For instance, the composition operation *(inside* ; *inside)* ; *inside* results in the set {*inside*, *on*, *outside*, *off*}, whereas the composition operation *inside* ; *(inside* ; *inside)* results in the set {*inside*, *outside*, *off*}. Some reasons that make the associative axiom fail are the behavioral differences between containers and surfaces. For instance, an assumption of this work has been that an object can be a surface for another object of the same size (e.g., two papers of the same size); however, an object cannot be a container for an object of the same or bigger size (e.g., such as two boxes of the same size cannot be put one into the other).

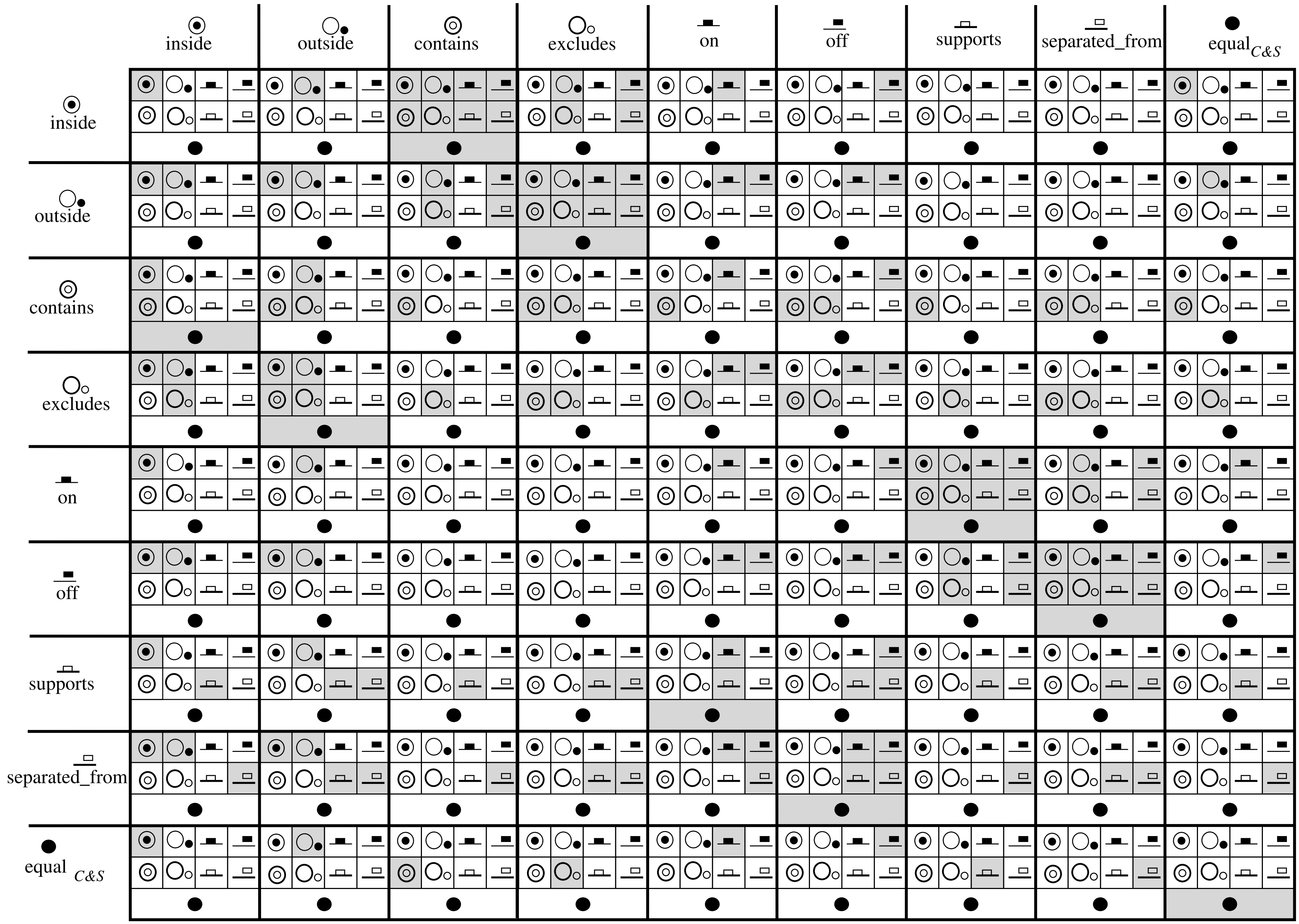


Table 3: Composition table of the container-surface algebra for objects playing only one role, being either a container or a surface (dark boxes denote possible inferred relations) (Egenhofer & Rodríguez in press).

When analyzing the composition of spatial relations within the room space, an important assumption has been that within a configuration objects play only one role, either as a container or as a surface. For example we exclude situations much like the desk in Figure 1, where a book may *have_off* another book that is *inside* a desk. While the desk is a container for the second book, it may also be a surface for the first one. In addition, the composition table reflects only spatial relations that have been derived from configurations involving containers and surfaces. Thus, it assumes that between two different objects, there is always one object that plays the role of a container or surface with respect to the other. For example, the composition table does not consider the composition of pen *inside* box ; box *contains* ball since between the pen and the ball—none of them being a container or a surface— no spatial relation can be derived from the container-surface algebra.

3.3 Inferences with the Container-Surface Algebra for Small-Scale Objects

Based on the container-surface algebra, a number of inferences can be made. These inferences must satisfy *consistency constraints* (Maddux 1990, Mackworth 1977) to avoid any contradictions among the spatial relations that describe a scene. Given a scene represented as a directed graph, in which nodes represent objects, directed edges represent binary spatial relations, and paths represent sequence of edges that follow a direction, consistency constraints can be formulated as a satisfaction of *path consistency* (Mackworth 1977). To guarantee consistency of compositions, the final set of possible relations between two objects must be derived from the intersection of all possible compositions that relate these two objects. The set of possible relations between objects i and j (R_{ij}), can be derived from Expression 3 (Egenhofer & Sharma 1993).

$$\forall_{i,j} R_{ij} = R_{ia};R_{aj} \cap R_{ib};R_{bj} \cap \ldots \cap R_{in};R_{nj} = \bigcap_{k=a}^{n} R_{ik};R_{kj} \qquad (3)$$

For an incomplete description of a scene, the process to infer unknown spatial relations may be defined for the following steps:

- Construct a node-consistent network, i.e., $\forall_i R'_{ii} = R_{ii} \cap equal$ and $\forall_{i,j|i\neq j} R'_{ij} = R_{ij}$
- Construct a arc-consistent network, i.e., $\forall_{ij} R''_{ij} = R'_{ij} \cap R'_{ji}$
- An iterative process that satisfies the path consistency (Expression 4). This iterative process ends when a new iteration does not produce any change in the spatial relations between any two objects.

$$\forall_{i,j} R'''_{ij} = \bigcap_{k=a}^{n} R''_{ik};R''_{kj} \qquad (4)$$

To illustrate this process, consider a database with three facts describing a scene: a pen is *on* a table, the table *supports* a box, and a room *contains* a table. Figure 2a represents this scene as a directed graph. To ensure node-consistency and arc-consistency, the equal and converse relations are added to the initial set of known spatial relations. From the composition operation and path-consistency constraints, possible inferences are:

- $R_{pen,box}$ = pen *on* table ; table *supports* box
  = pen *inside* box xor pen *outside* box xor pen *on* box xor pen *off* box xor pen *contains* box xor pen *excludes* box xor pen *supports* box xor pen *separated_from* box xor pen $equal_{C\&S}$ box
  = $U_{C\&S}$

- $R_{\text{box,pen}}$ = box *on* table ; table *supports* pen
  = $U_{C\&S}$
- $R_{\text{room,box}}$ = room *contains* table ; table *supports* box
  = room *contains* box
- $R_{\text{box,room}}$ = box *on* table ; table *inside* room
  = box *inside* room
- $R_{\text{pen,room}}$ = (pen $U_{C\&S}$ box ; box *inside* room) ∩ (pen *on* table s table *inside* room)
  = (pen *inside* room xor pen *outside* room xor pen *contains* room xor
  pen *excludes* room xor pen *supports* room xor pen *separated_from* room xor
  pen $equal_{C\&S}$ room) ∩ pen *inside* room
  = pen *inside* room
- $R_{\text{room,pen}}$ = (room *contains* box ; box $U_{C\&S}$ pen) ∩
  (room *contains* table ; table *supports* pen)
  = room *contains* pen

The final directed graph (Figure 2b) showed that refinements of the composition operations can be made. A pen is not the same as a box, therefore, the relation $equal_{C\&S}$ is impossible between the box and the pen. Since it is known that a pen can be neither a container nor a surface, we can discard the possible spatial relations that imply one of these roles for the pen. If the pen is smaller than the box, we can reduce the possible spatial relations between the pen and the box to pen *inside* box xor pen *outside* box.

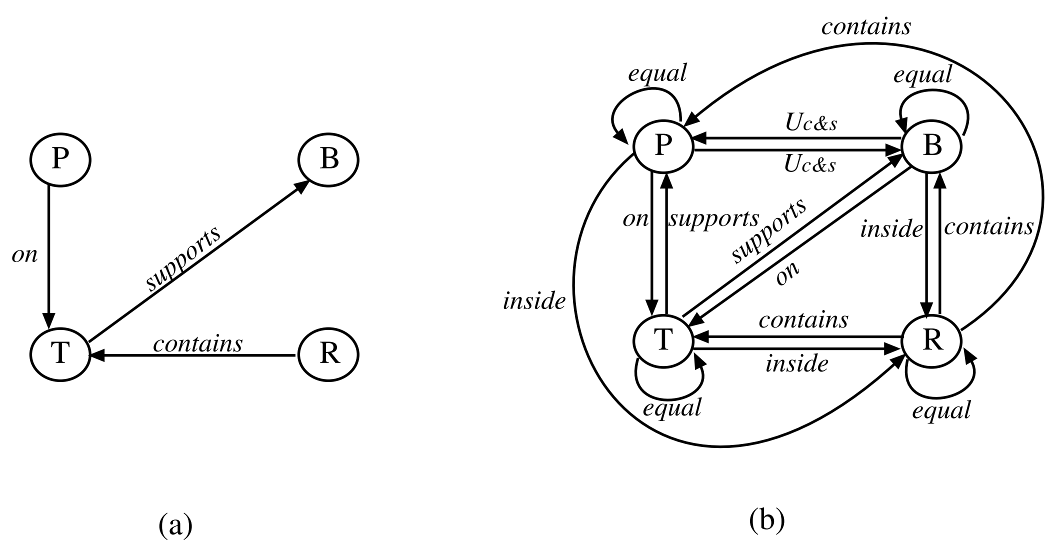


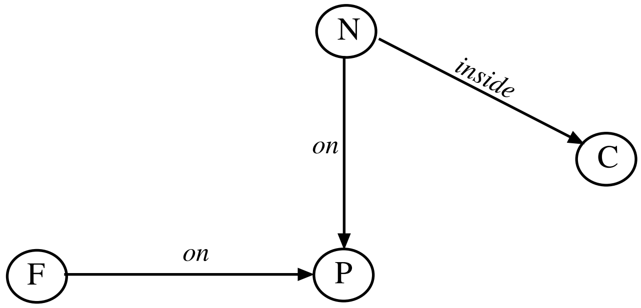
Figure 2: Inferences with small-scale objects: (a) initial scene description and (b) completed scene description that satisfies node-, arc-, and path-consistency constraints (P is a pen, B is a box, T is a table, and R is a room).

4. The Container-Surface Algebra for Large-Scale Space

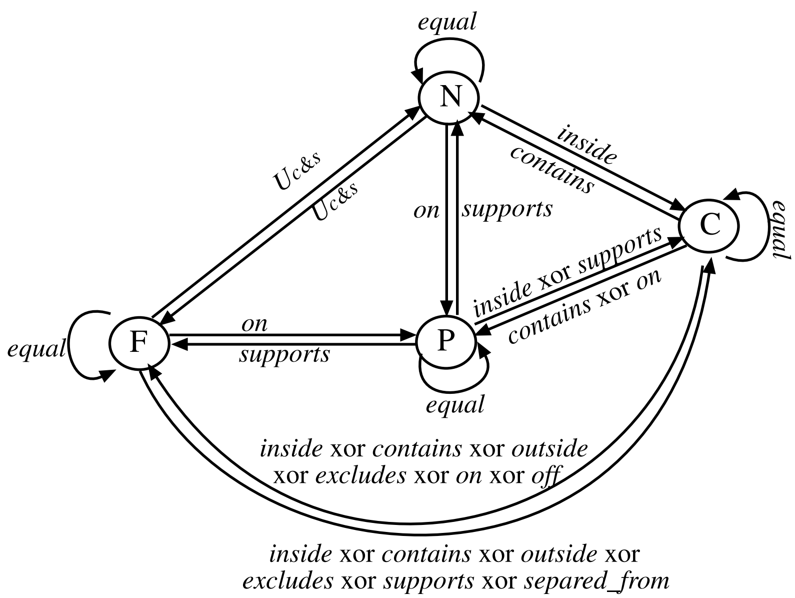
Variations in the types of objects—solid vs. liquid (Hayes 1990)—their nature—physical vs. administrative or *fiat* objects (Smith 1995)—or their sizes—microscopic vs. table top vs. large-scale space (Zubin 1989, Montello 1993)—may affect the coherence of the container-surface algebra. The basic properties of the container-surface algebra were derived from a setting with manipulable objects. While such a setting corresponds to the theory of acquiring image schemata through bodily experiences, it would be limiting if the applicability of the algebra was restricted to settings with the same properties. To explore the application range and consistency of the container-surface inferences, we analyzed different configurations that comprise different spatial objects with respect to their type, nature, and sizes. Using objects of different sizes could lead to different senses of spatial relations (Herskovits 1986), such as a luggage is inside of a container and a lake is inside of a county. Thus, this analysis may indicate whether the basic behavior of image schemata can be basis for the integration among different geometric conceptualizations of spatial relations. Like in the small-scale space, selected configurations combine the spatial relations *inside* and *on* for situations between surfaces and containers. Then, inferences were applied as they did apply in the small-scale space to complement the possible spatial relations between objects.

4.1 Inferences with Large-Scale Objects

In a first instance, configurations with only large-scale objects were analyzed. A set of large-scale objects, a mountain, a national park, a county, a peninsula, a forest, and a lake, was selected. Like in the room space, these objects were combined to provide all possible combinations between surfaces and containers. Figure 3a presents a partial scene description, as a directed graph, with known relations among these objects. The scene after applying composition operations and consistency constraints is shown in Figure 3b. For this scene, it is also possible to make further refinements of the inferences when information about relative sizes and roles of the objects is considered. For example, if it is known that the peninsula is smaller than the county, the only possible spatial relation between them would be that peninsula is *inside* the county and, therefore, the forest would be also *inside* the county.



(a)



(b)

Figure 3: Inferences with large-scale objects: (1) initial scene description and (b) completed scene description that satisfies node-, arc-, and path-consistency constraints (N is a national park, F is a forest, P is a peninsula, and C is a county).

4.2 Reasoning with Small-Scale and Large-Scale Objects

The next analysis considers configurations with a combination of objects belonging clearly to different scales. Following the same approach to the large-scale space, six objects (entities) were selected: passenger, luggage, runway, container, airplane, and airport. An initial scene with a subset of known relations that is presented as a directed graph is shown in Figure 4a. The final scene after applying composition operations and consistency constraints is shown in Figure 4b. Like in the previous cases, further refinement of the inferences can be made as we consider sizes and roles of the objects within the scene. For example, knowing that the runway is smaller than the airport, we can infer that the runway can only be inside the airport.

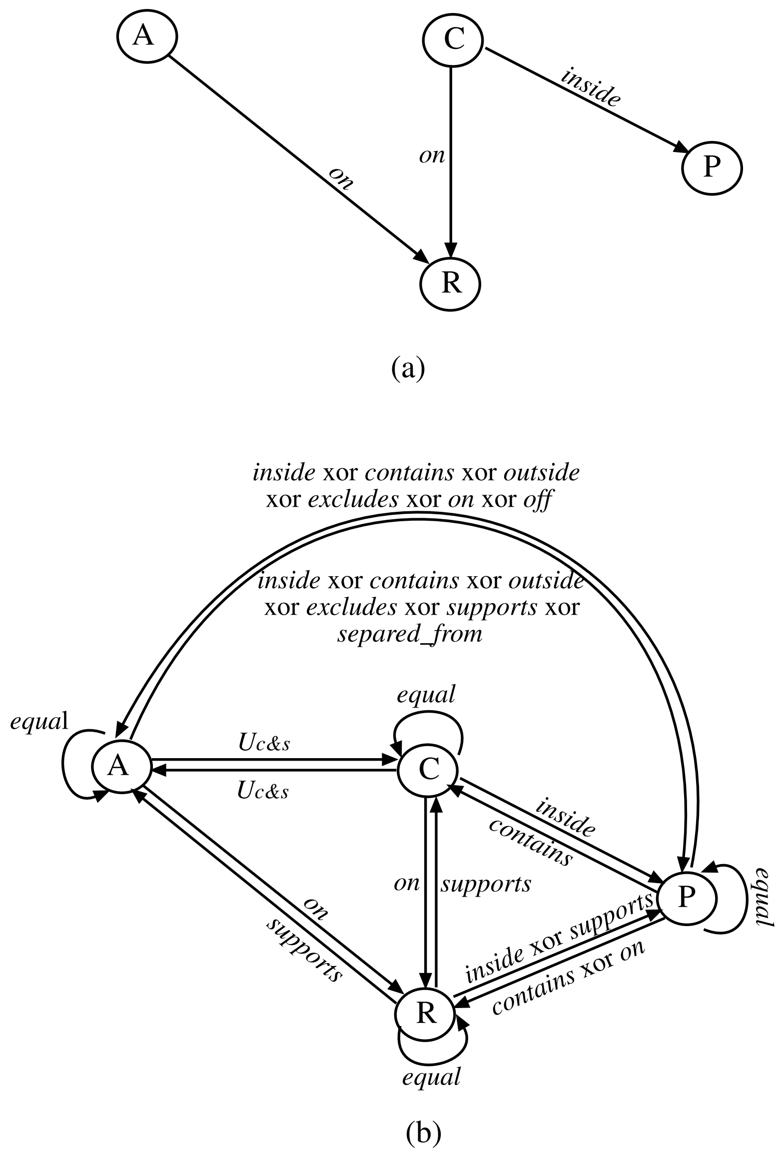


Figure 4: Inferences with small- and large-scale objects: (a) initial scene description and (b) completed scene description that satisfies node-, arc-, and path-consistency constraints (A is an airplane, C is a container, R is a runway, and P is an airport).

5. Evaluation of Inferences

To determine whether the inferences of the container-surface algebra correspond to the way people reason about spatial relations, we performed a human-subject study to compare people's inferences with those of the container-surface algebra. We were particularly interested in evaluating the applicability of the algebra across scales and detecting whether people's inferences match with the properties of the compositions operations.

5.1 Survey

We designed a survey that presented three different scenes described a sets of facts such as:

- The lake is *inside* the national park.
- The national park is *inside* the county.
- The forest is *on* the mountain.
- The mountain is *on* the peninsula.
- The national park is *on* the peninsula.

No pictorial clues, such as a map of the setting, were given. Subjects were then asked to name the relations between given pairs of objects, such as

- The lake ___________ the county.

Subjects needed to derive these relations by combining predicates, although each questionnaire included one case in which the answer was given by a single fact. One inference per set could not be determined by the algebra since the inference associates two objects that can be classified as neither container nor surface. The inferences represent different levels of difficulties by considering single as well as double compositions, and compositions that produce unique or ambiguous results. The subjects were instructed to answer “no relation” where they believed no relation between the objects exists. In order to allow the subjects to answer the survey within 15 minutes, only a representative subset of all possible inferences was asked. Each survey contained three scene descriptions, one describing a small-scale space, the second a large-scale space with large-scale objects, and the third described small-scale and large-scale objects embedded in a large-scale space.

Table 4 shows the set of compositions and the reference result from the container-surface algebra. Refinement by relative size and role of objects was applied to eliminate unrealistic answers. For example, although the composition *supports* ; *inside* results in two possible answers (*inside* xor *supports*), the relation *supports* was not considered as correct for the composition of table *supports* box ; box *inside* room, because rooms and airports are bigger than tables. Refinements by size led to different inferences depending on the particular setting.

Configuration	Composition	Inferred Relations
Small Scale		
ball-room	*inside* ; *inside*	inside
pen-table	*on* ; *on*	*on*
ball-table	*inside* ; *on*	*on*
table-room	*supports* ; *inside*	*inside*
pen-room	on ; *supports* ; *inside*	*inside*
paper-room	*on* ; *supports* ; *inside*	*inside*
pen-ball	undefined	no relation
Large Scale		
lake-county	*inside* ; *inside*	*inside*
forest-peninsula	*on* ; *on*	*on*
lake-peninsula	*inside* ; *on*	*on*
peninsula-county	*supports* ; *inside*	*inside*/*supports*
forest-county	*on* ; *supports* ; *inside*	*inside*/*outside*
mountain-county	*on* ; *supports* ; *inside*	*inside*/*outside*/*supports*/*separated_from*
lake-forest	undefined	no relation
Mixed Scale		
luggage-airport	*inside* ; *inside*	*inside*
passenger-runway	*on* ; *on*	*on*
luggage-runway	*inside* ; *on*	*on*
runway-airport	*supports* ; *inside*	*inside*
passenger-airport	*on* ; *supports* ; *inside*	*inside*
airplane-airport	*on* ; *supports* ; *inside*	*inside*
passanger-luggage	undefined	no relation

Table 4: Survey description: compositions and reference inferred relations from the container-surface algebra (/ denotes exclusive or).

Thirty-eight students of two undergraduate classes at the University of Maine participated in the survey. For the analysis, only those 30 students were considered whose mother tongue is US English. We recorded the subjects' gender (1/3 female, 2/3 male) and found no significant differences between their responses. The ordering of the scene and facts for each scene was changed among the surveys to evaluate its effect on the answers. There were also no significant differences due to the different orderings of the questions posed.

5.2 Results

Out of a total of 510 expected answers that are defined in the container-surface algebra, 342 matched exactly with the inferences of the relation algebra (67%), and another 47 (9%) gave a subset of the possible correct answers. Two questions were not answered (0.3%). In 71 cases (14%), a relation other than the predicate names used in the relation algebra was given. Only 10 answers (2%) had a relation that was part of the predicate names used in the relation algebra, but different from the correctly inferred relation. In 15 cases (3%), subjects stated that there was no relation, although the relation algebra would infer one. Of the 90 cases that had multiple possible answers, "no relation" was given in 23 answers (26%). None of subjects used a relation defined within the container-surface algebra to describe the 90 cases that were undefined by the algebra.

In counting matches between the subjects' answers and the reference results we considered as correct answers those that coincide with the reference answer of the container-surface algebra or those that were a subset of correct answers for questions with more than one possible result. Table 5 shows the distribution of the matches between the algebra and the subjects' answers for each question.

Configuration	Answers	Number of correct answers	%
Small Scale			
ball-room	19	19	100 %
pen-table	30	29	97 %
ball-table	30	29	97 %
table-room	30	28	93 %
pen-room	30	27	90 %
paper-room	30	27	90 %
Large Scale			
lake-county	21	20	95 %
forest-peninsula	30	27	90 %
lake-peninsula	30	22	73 %
peninsula-county	30	12	40 %
forest-county	30	19	63 %
mountain-county	30	17	57 %
Mixed Scale			
luggage-airport	20	14	70 %
passenger-runway	30	28	93 %
luggage-runway	30	27	90 %
runway-airport	30	14	47 %
passenger-airport	30	16	53 %
airplane-airport	30	14	47 %

Table 5: Number of answers that match reference results.

We defined four hypotheses to be evaluated by statistics tests. While the first hypothesis checks every individual question, the rest three hypotheses group the question by scale, type, and complexity.

Hypothesis 1: People evaluate all inferences according to the container-surface algebra, independent of the scale of the objects and the embedding space.

This hypothesis was rejected. By using a normal approximation to the binomial distribution, individual questions were analyzed to test whether the probability of correct answers is equal to 0.75 (null hypothesis) with respect to the alternative hypothesis that the probability is different from

0.75. The results suggest that for small-scale spaces the algebra matches with people's reasoning about spatial relation (Table 6). For small-scale spaces the probability that the algebra gives the correct answers is over 0.75, reaching for some cases probabilities of over 0.90, with a level of significance of 0.05. For configurations with large-scale objects only, there were two out of six inferences that deviated significantly from the algebra. Similarly, in mixed-scale space, three of six inferences were significantly different from the subjects' assessments.

Configuration	Answers (n)	Observation (o)	Test $-1.607 < Z = \frac{(o - n*0.75)}{\sqrt{n*0.75*0.25}} < 1.607$	Conclusion
Small Scale				
ball-room	19	19	2.527	p > 0.75
pen-table	30	29	2.741	p > 0.75
ball-table	30	29	2.740	p > 0.75
table-room	30	28	2.319	p > 0.75
pen-room	30	27	1.897	p > 0.75
paper-room	30	27	1.897	p > 0.75
Large Scale				
lake-county	21	20	2.142	p > 0.75
forest-peninsula	30	27	1.897	p > 0.75
lake-peninsula	30	22	-0.211	p = 0.75
peninsula-county	30	12	-4.427	p < 0.75
forest-county	30	19	-1.476	p = 0.75
mountain-county	30	17	-2.319	p < 0.75
Mixed Scale				
luggage-airport	20	14	-0.516	p = 0.75
passenger-runway	30	28	2.319	p > 0.75
luggage-runway	30	27	1.897	p > 0.75
runway-airport	30	14	-3.584	p < 0.75
passenger-airport	30	16	-2.741	p < 0.75
airplane-airport	30	14	-3.584	p < 0.75

Table 6: Test (H_0: p = 0.75) for individual questions.

Hypothesis 2: People make the same kinds of inferences in small-scale and in large-scale space.

This hypothesis was rejected. We used a chi-squared distribution for a non-parametric test, i.e., a test that does not consider parameters of the distribution, to evaluate whether the survey suggests any conclusion about the correctness of answers taking questions as groups of small-scale, large-scale, and mixed-scale spaces. The null hypothesis was that the number of observed correct answers is equal to the number of expected correct answers. Every test used a probability of 0.05 that we reject the null hypothesis when in fact it is true.

As expected from the first test, the small-scale inferences were accepted, whereas the large-scale and mixed-scale inferences were rejected (Table 7).

Configuration	**Test** $\chi^2 = \sum_{i=1}^{6} \frac{(o_i - \pi_i)^2}{\pi_i} < 11.07$	**Conclusion**
Small scale	0.8	accepted
Large scale	22.9476	rejected
Mixed scale	25.8333	rejected

Table 7: Test for groups of questions based on scale.

Hypothesis 3: The agreement of people's inferences with the container-surface algebra depends on the complexity of the composition.

This hypothesis was accepted. This test also used a non-parametric test with a chi-squared distribution to assess whether there are significance differences between the observed answers and the expected answers for inferences that represent (1) transitive compositions, (2) binary but non-transitive compositions, and (3) ternary and higher compositions that were not transitive. Analyzing the answers by class of composition and number of inferred relations, it is clear that the transitive property is suggested in all cases (Table 8). Binary compositions such as *inside* ; *on* that give only one answer are likely to be consistent with people inferences. Double and tripe compositions such as *on* ; *supports* ; *inside* and *on* ; *on* ; *supports* ; *inside* present major difficulties to derive correct answers. This observation is consistent with the fact that the container-surface algebra is non-associative and its results are given by the intersection of both possible lines of reasoning.

Configuration	**Test** $\chi^2 = \sum_{i=1}^{6} \frac{(o_i - \pi_i)^2}{\pi_i} < 11.07$	**Conclusion**
Transitive compositions *inside* ; *inside* and *on ; on*	2.313	accepted
Binary composition *in* ; *on* and *supports ; inside*	21.934	rejected
Ternary composition *on* ; *supports ; inside*	25.333	rejected

Table 8: Test for groups of questions based on type of composition.

Hypothesis 4: The agreement of the people's inferences with the container-surface algebra depends on the complexity of the result of the composition.

This hypothesis was accepted. Like hypotheses 2 and 3, we used a non-parametric test with a chi-squared distribution to evaluate whether observed answers match the expected answers for groups of questions classified by the number of inferred relations. For compositions that result in multiple answers people do not infer any relation or give wrong answers (Table 9).

Configuration	Test $\chi^2 = \sum_{i=1}^{9} \frac{(o_i - \pi_i)^2}{\pi_i} < 15.5$	Conclusion
Single result *inside* ; *inside* *on ; on,* *inside ; on*	4.781	accepted
Multiple results *supports ; inside* *on ; supports ; inside*	44.8	rejected

Table 9: Test for groups of questions based on number of inferred relations.

5.3 Discussion

The small number of incorrect inferences within the domain (10 out of 510) is evidence that the basic principles of the container-surface algebra—converseness of relations and the composition operation to capture the interplay between containers and surfaces—match with human intuition.

Based on the algebra, inferences from the small-scale and mixed-scale configurations should give crisp results (i.e., compositions without any ambiguity), since the objects' relative size is implicit and a refinement of the algebra can be done for compositions with possible multiple results. The large scale-space scenario, however, includes 3 inferences with single result and 3 inferences that have multiple possible result. Subjects gave 341 correct answers for inferences of crisp results, representing 81% of the expected number. These 341 correct answers correspond to 99% of the total of 342 correct answers obtained from the survey. Thus, it is clear that the complexity of result has a great effect on the subjects' answers. In a few cases (15), the algebra actually provides more crisp results than the subjects' inferences. On the other hand, the algebra considered a larger set of answers (i.e., additional relations the subjects did not consider) in 47 cases.

The number of inferences outside of the domain of the container-surface algebra (71 out of 510) were largely due to the use of the prepositions *at* (32 times) and *part of* (7 times)—with another two terms (i.e., near and leaves) used four times, four terms (i.e., above, overlap, divide, and by) used twice, and sixteen terms (e.g., includes, surround, around, fills, over, and next) that were used once. The preposition *at* was used only in the context of the mixed-scale scenario, and there in inferences with expected answer *inside*. When considering preposition *at* as a synonym of *inside*, the statistical analyses for mixed-scale space accept both the first and second hypothesis. This deviation from the other references can be explained as a linguistic choice of predicates, as explained by Herskovits (1986) where *at* is defined as "a point to coincide with another." Although the reference, such as the airport, and the located objects, such as the passenger, are not actually points, they are viewed as such. In a later work, Herskovits (1997) extends her definition by stating that *at* is "coincidence of a movable point objects with a point place in a cognitive map." Hence, *at* can not be used in large-scale spaces. The mixed-scale scenario could be interpreted in such a way that the airport does not necessarily surround the passenger and it is considered as a point in the cognitive map. Thus, the preposition *at* better represents the configuration between passenger and airport.

The relation *part* was used in both large-scale space (5) and mixed-scale space (2). From the 7 times when *part* was used only once it was used when the expected result was *on*. Winston *et. al.* (1987) discussed the usual confusion between meronymic relation (*part of*) and the topological inclusion. In some cases, meronymy involves a spatial inclusion, since a located object may completely overlap the reference object and at the same time be part of the reference object (for example, a peninsula that is *inside* of a county and it is at the same *part of* the county).

6. Conclusions and Future Work

This study defined a spatial-relation algebra with a small set of spatial operators (*inside, on, outside, off, and their respective converse relations*). The container-surface algebra provides an inference mechanism to derive spatial relations from the composition of individual as well as combinations of image schemata.

The human-subject testing suggests that spatial inferences derived from the container-surface algebra seem to be sensible for small-scale configurations; however, the applicability of this algebra may require an adaptation to configurations with objects of large or mixed sizes. Future studies should analyze whether there exists another set of spatial relations that better describe large-scale or mixed-scale configurations. For example, the distinction between *inside* and *at* needs further investigation. Likewise, *part of* is usually confused or combined with the spatial relation *inside* and *on*. The complexity of the composition (i.e., the combination of spatial relations) and the results of the composition (i.e., the number of possible inferences) affects the agreement of the people's inferences with the container-surface algebra. Transitive compositions, such as *inside* ; *inside*, are consistent with people's judgment. Compositions with different spatial relations, such as *on* ; *supports* and *inside* ; *on*, give consistent results when there is only one possible inference.

An area for further investigation is to explore how the spatial-relation algebra is affected by incorporating the *part-whole* image schemata. The part-whole schema appears to be an important factor for discriminating spatial relations. What portion of an object needs to be *inside* or *on* another object to consider the object *inside* or *on*, respectively? Is shape and predominance of a portion relevant for distinguishing if an object is *inside* or *on*?

The container-surface algebra constitutes an alternative approach to spatial reasoning. Thus, a further study should confirm or dismiss whether these inferences match with any of the inference made with the traditional spatial reasoning approaches. Do the different approaches provide complementary answers? Can we map the container-surface algebra onto an algebra that uses topological or geometric properties?

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